CLASSIFIED: **BOTTOM SECRET**

Playnet

literally an actual alternative to capitalism

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contact: momo-nozomi@riseup.net

Let us finally imagine a **Free and Equal Association of United Producers** carrying on their work with the **means of production** held in *common*, in which the **labour power** of all the different individuals is *consciously applied* as the **combined labour power** of the *community*.

1. Desire, possibility, tension, and harmonious relaxation

The whole system in one view, before the architecture that follows builds it piece by piece.

One field. Over every possible allocation of the federation's labour lies a single field of **tension** - the total, in labour-hours, by which a configuration sits away from where the federation wants to be. The plan works to **relax** it; **harmony** is the concord that relaxation achieves - the well-tempered whole, measured as tension resolved:

$$\underbrace{\mathcal{H}}_{\text{harmony}} = -\underbrace{\sum_{d} D_d}_{\text{desire-tension}} - \underbrace{\sum_{a} P_a}_{\text{possibility-tension}}$$

The plan is the allocation that **maximises harmony** - that most fully **relaxes tension** - the way a piece of music builds and resolves its tensions, struck beat by beat. Harmony is tension well-resolved.

Harmony reconciles desire with possibility. Every term is a tension: a continuous, labour-time measure of distance from where we want to be. They come in two species, the two things harmony has always been reconciling.

> **Desire-tension** $D_d = w_d^{\text{desire}} \cdot g_d^2$ - the unmet want on a final good d, the squared shortfall weighted by social desire. It is **soft**: bounded, so desires trade against one another - under-serve cake to serve medicine. It pulls allocation **toward producing more**. The voice of *what we want* - the realm of freedom.
>
> **Possibility-tension** P_a - the strain of pressing against a limit of the possible: labour endurance ($L/A \to 1$), capacity, skill redundancy, reserves, the ecological envelopes, care. It is **hard**: it diverges as the state nears its tipping point - and that divergence is **fragility** - so near the limit

it out-pulls everything and is never crossed. It pulls allocation **away from the edge**. The voice of *what is possible* - the realm of necessity.

A high possibility-tension means possibility nearly **spent** - the watershed strains as the draw nears its envelope. A binary gate - a certified surgeon, material balance - is its step-function extreme.

Where the tensions come from. Desire-tension is built bottom-up: a citizen's priority weights over the desire tree, scaled by consumer voice and aggregated, give social desire $w_d^{\text{desire}} = \sum_p v_p^{\text{cons}} \cdot \hat{x}_p(d)$. Desire enters only at the final-good layer; nobody weights an oven. **Possibility-tension** is the voice of the materially possible: each limit a continuous barrier sized by the **computation of probabilities** - variability, lead-time risk, distance-to-tipping - the speech of producers, technique, and nature.

The plan is the resolving chord - forever resolving, never resolved. It minimises total tension - equivalently, the configuration where **every tension pulls equally hard**. The **pull** of a tension is the marginal harmony that relaxing one more unit of it would yield. At the optimum, the **desire-pull** of one more unit produced equals the **possibility-pull** it incurs. Soft desire-tensions trade freely against one another; hard possibility-tensions hold their limits inviolably, so an ecological cap simply out-pulls any desire that nears it. It is one field, one currency - the chord less a thing that lands than the next voicing in an unending progression.

Harmony is *metabolised* - it emerges from the parts in continuous local feedback, struck beat by beat. The pull is the ideal; the cascade is how the parts realise it. A starved high-priority desire keeps a **loud** gap - the quadratic curvature makes an unmet high-desire deficit grow steep - and that loudness propagates upstream through the recipe graph: a humble oven-repair inherits the loudness of the bread waiting behind it. Derived demands carry no desire of their own; they are pulled exactly as hard as what they hold up. The cascade of loud gaps is the distributed realisation of the balance of pulls.

Recomposed in real time. The loud pulls steer the recipes. Where a possibility-pull runs high - a scarce input, a strained capacity - the plan recomposes onto alternative recipes; one that stays high and cannot be relieved quickly escalates into a reserve-draw or a capacity investment. Where a desire-pull runs high, labour flows toward it. The live plan is this continuously recomposed configuration - the chord re-voiced, beat by beat, to keep tension relaxing toward harmony.

The substrate is labour-time. Every tension and every pull is denominated in labour-hours. The surface shows socially necessary direct labour time $\text{SNLT}_{\text{direct}(r)}$ (living labour, where it is spent, summing to L); credit and recipe-sizing use socially necessary crystallized labour time $\text{SNLT}_{\text{crystallized}(r)}$ (the total congealed in a unit). One currency, so the whole field is commensurable and everything is internal.

A living practice. The federation values its own tensions in the open - in labour-hours, under governance - and relaxes them into harmony moment to moment, the way a living body holds homeostasis or a piece of music builds and resolves its tensions. Coherence is **metabolised**: a practice, perpetually re-struck. This is the third thing past the market and the central plan - it reaches coherence through open valuation and continuous relaxation, where the one leaves it to exchange and the other to forecast.

The sections that follow build this architecture from the ground up - the loop from desire to plan, labour and free time, desire and its pull, the surface and its categories, governance and the two measures of

labour, then credit and distribution, the federation, and the living plan it runs - and return, at their close, to harmony.

2. Labour, skills, and the surface

2.1. From desire to plan

Citizens express their desires as priority weights over needs - a shape spanning the entire field of **means of consumption**. These individual shapes are weighted by **consumer voice** v_p^{cons} and aggregated into a **social desire**: the federation-level demand that the plan must try to satisfy.[1]

An automated planner proposes plans - each a complete assignment of labour to scopes (branches of productive activity, each with its own governance), recipes to resource specifications (the kinds of thing produced), and outputs to the distribution rails. The plan is organised on the basis of the technical-material requirements of the production process, which are determinable by engineering and ecology. The scope surface of *The surface* below (Figure 2) is one such plan among many the planner could generate. Citizens weight competing proposals through their **producer voice**; the accepted plan is the one that best solves for harmony of the social desire.

2.2. Labour, skills, and free time

The federation's citizens each carry a **skill set** drawn from the federation's skill catalogue. A citizen **offers availability** - time windows, intensities, spatial catchments[2] - declaring when and where they can work. The planner issues **work-intents**: a demand for a specific skill at a specific time in a specific catchment. A match occurs when the citizen holds the required skill and their offered window intersects the demand.

The hours a citizen is matched to become **validated labour** - the matching and the ledger are directly connected. The social judgment lives **upstream** in *plan acceptance*; once a plan is accepted and you are matched to its work-intents, those hours are simultaneously your work, your voice, and your credit. The **federation aggregate** is the sum of every citizen's matched hours - the federation's total social labour.

$$\underbrace{L}_{\text{total labour}} = \sum_{p} \underbrace{L_p}_{\text{a citizen's hours}}$$

Each **scope** - a branch of productive activity with its own governance (food production, manufacturing, education…) - carries its own total: the sum of matched hours within that scope is the scope labour. Over a **planning horizon** (hours per citizen), the **social free time** is the share of total social time the solved plan leaves unplanned: one minus the plan's planned hours Λ divided by the product of citizens and hours each. Λ is the labour the plan places (making, moving, training), read

[1] Marx, *Capital* Vol. III, Ch. 49: "Production under the actual, predetermining control of society establishes a relation between the volume of social labour-time applied in producing definite articles, and the volume of the social want to be satisfied by these articles."

[2] A spatial catchment is a geographic **catchment cell** in the federation's spatial index - a neighbourhood, a district, a region (a place region only; distinct from the surface's treemap cells and the lattice's spacetime cells introduced later). Work-intents specify a catchment when the work must be performed at a particular location (a clinic, a farm, a construction site). Citizens offer availability within catchments they can reach; the matching places them only into work-intents whose catchment intersects their offer.

off the solve when it runs and published with it; the validated hours L are what has been worked, and they set each person's share, L_p/L, and never f.

$$\underbrace{f}_{\text{free-time fraction}} = 1 - \frac{\overbrace{\Lambda}^{\text{hours the plan places}}}{\underbrace{N \cdot T}_{\text{citizens} \times \text{hours each}}}$$

As the **productive forces** (means of production, labour-power, skills, scientific knowledge, and social organisation) grow, social free time rises toward its maximum.[3]

Behind free time lies a plainer identity. A society's **total social time** - every waking hour of all its people - divides in two: the socially necessary labour that reproduces its conditions, and the free time left above it.

$$\mathrm{TST} = \mathrm{SNLT} + \mathrm{SFT}$$

As the productive forces develop, the SNLT required to hold those conditions in being **falls** toward an irreducible residue - something must always be watched, inspected, kept - and $\mathrm{SFT}/\mathrm{TST} \to 1$: the realm of necessity contracting toward its floor, the realm of freedom opening above. The wealth of such a society is then read not as its output but as the free time it has won - the hours its people are free to **be**, once the conditions of their being are reproduced.

Two measures read the federation's state against this identity. The social free time f above is one. The other starts from each citizen: every citizen carries a **satisfaction level** σ_p between zero and one, the degree to which their weighted desires are met (the weights are priority preferences over desires, set out in *Desire, pull, and floor* below). The **social desire satisfaction** Φ is the harmonic mean of every citizen's satisfaction, so it collapses toward zero whenever one citizen's is near zero - it is structurally sensitive to whoever is worst off. The **social play time** $\mathcal{P} = \min(f, \Phi)$ is the lesser of the two: it approaches its maximum only when most of social time is free **and** every citizen's desires are fully and equally satisfied. This joint limit is what the federation moves toward. The solver turns its crank on harmony the appendix *Why the objective can be trusted* measures how closely harmony tracks social play time, and shows where the two part company and why that parting is a political decision.

2.3. Vocabulary: ValueFlows and the two labour times

Production in the federation is described using ValueFlows (VF), a vocabulary for distributed economic networks built on three layers:

> **Knowledge.** **Resource specifications** define **what kind of thing** something is - the lowest-level type (a specific bread, a specific vaccine, a specific tool). Above them, **resource classifications** form a hierarchy of categories: food → grain → bread. A **recipe** is a reusable template for producing or maintaining a resource: its inputs, the **processes** that transform them, and the outputs they yield - a directed graph from raw ingredients to finished product. A resource may have both a **production recipe** (build a house) and a **maintenance recipe** (maintain a house for one period); both generate work-intents in the plan.

[3] Marx, *Grundrisse* (Notebook I): "Economy of time, along with the planned distribution of labour time among the various branches of production, remains the first economic law on the basis of communal production. It becomes law, there, to an even higher degree."

Plan. The planner generates **plans** from recipes by scaling to demanded quantities. Each plan carries **work-intents** (demands for specific skills at specific times). When a citizen's availability matches and they agree, an intent becomes a **commitment** - a scheduled promise.

Observation. Economic events record what actually happened - resources produced, consumed, or transformed. Events are immutable: they describe the past.

Each resource specification carries two **socially necessary labour times**. Its **socially necessary direct labour time** $\mathrm{SNLT}_{\mathrm{direct}(r)}$ is the **living** labour added at its own stage - the baking in a loaf, the milling in flour - to (re)produce one unit. Its **socially necessary crystallized labour time** $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$ is the **total** labour congealed in the unit: the direct labour at this stage, plus the crystallized labour of every input consumed, plus the wear of every means of production used, rolled up recursively over the recipe from raw material to finished good. The two do different work. The surface renders the **direct** measure, so every hour appears once, where it is spent, and the areas sum to L; credit and recipe-sizing use the **crystallized** measure, what a claim costs society. *Reproduction labour and the two measures of labour*, below, develops the rollup, depreciation, repair, and containment.

2.4. Desire, pull, and floor

Citizens express desire through the same resource classification hierarchy that structures production - food → grain → bread. At each branching point, the citizen sets **priority weights** among siblings: food 0.30, housing 0.25, clothing 0.10. These weights express **relative importance** - they span both the individual-claimable and communal rails of the means of consumption. On the individual-claimable rail, priority weights shape the citizen's order book - what they draw against their credit. On the communal rail, the same weights carry the citizen's voice into collective allocation where no credit is spent.

Where the citizen sets no weight, the default is the previous aggregate demand signal. A citizen who touches nothing accepts the current collective priorities; a citizen who adjusts food upward shifts at exactly one branching point. Across the **futures** a plan is held in, the same weights are judged without a probability: each future's captured share of its own achievable value, the plan lifting the least of them (the maximin reading under *Reproduction under uncertainty*).

Weights multiply down the path. The citizen's **priority weight** on a leaf desire is the product of every weight along the resource classification path from root to leaf.

$$\underbrace{\hat{x}_p(d)}_{\text{priority weight on leaf } d} = \prod_{i \in \text{ path}(d)} \underbrace{w_{p,i}}_{\text{weight at branch } i}$$

For example, a citizen's path to sourdough bread:

$$\overbrace{w_{\text{food}}^{0.30}}^{\text{MoC}} \times \overbrace{w_{\text{grain}}^{0.50}}^{\text{food}} \times \overbrace{w_{\text{bread}}^{0.40}}^{\text{grain}} \times \overbrace{w_{\text{sourdough}}^{0.60}}^{\text{bread}} = \hat{x}_p(\text{sourdough}) = 0.036$$

One tree, two spaces: in **labour-hour space** the planner allocates social labour to satisfy social desire; in **desire space** the citizen sets priorities. The structure is shared; the proportions differ.

Each citizen's priority weights are weighted by their consumer voice and aggregated into a single **social desire** weight per dimension: the sum, over all citizens, of each citizen's consumer voice times their priority weight on that dimension.

$$\underbrace{w_d^{\text{desire}}}_{\text{social desire on } d} = \sum_p \underbrace{v_p^{\text{cons}}}_{\text{consumer voice}} \cdot \underbrace{\hat{x}_p(d)}_{\text{citizen's priority weight}}$$

The aggregate desire feeds the planner, which generates a concrete plan - recipes, processes, work-intents - that allocates social labour to satisfy it. crystallized SNLT is a derived quantity: the labour-content per unit of a resource, computable from the plan. **The citizen sets relative priorities; the planner produces a plan; labour is allocated within that plan.**

Every dimension of the social desire is scored by one curve (Figure 3): a shortfall costs quadratically, so scarcity is spread as many shallow cuts; a surplus earns nothing, so a goal is satiation and never a floor.

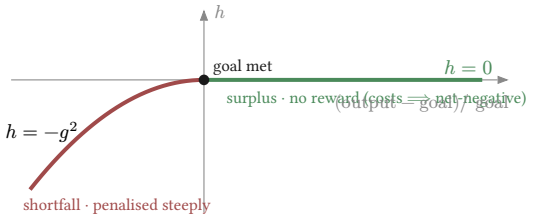

Figure 1: **The desire score *h*.** Each demand dimension is scored by how far output sits from its goal q_d^*. **Shortfalls** (left, red) open a gap $g_d = (\text{goal} - \text{output})/\text{goal}$ and are penalised *quadratically* - $h = -g_d^2$ - so deep cuts hurt disproportionately and the planner spreads scarcity as many shallow shortfalls rather than a few deep ones. **Surpluses** (right, green) earn *nothing* - $h = 0$, flat: a goal is *satiation*, not a floor, so "more" is never automatically better. And because surplus still **costs** (its crystallized SNLT keeps pulling on scarce labour and draw), the full pull past the goal is *net-negative* - the planner stops *at* the goal and sends the freed labour to the next unmet desire. Any wanted buffer lives *inside* the goal q_d^* - it is desire, scored here; the **insurance reserve** q_r^{res} is a separate **hard** tension, not a surplus reward. The Score sums $w_d^{\text{desire}} \cdot h(g_d)$ over all dimensions.

2.4.1. Derived demand and the pull of a bottleneck

Citizens desire bread, not the oven that bakes it. The means of production, the intermediate goods, the trained hands - none is wanted for itself; each is wanted only **through** the final goods it makes possible. This has a precise consequence for how harmony treats them.

Only **final desires** carry a desire-tension, weighted by consumer voice. A means of production carries no desire of its own; it carries a **possibility-tension** - bread output is limited by oven capacity, by the flour on hand, by the count of trained bakers. So the urgency of repairing an oven is never **assigned** - it is the oven's **possibility-pull**: the marginal harmony that one more unit of oven capacity would resolve.[4]

This **derived pull** self-corrects in every case a hand-coded rule would get wrong. An oven with slack capacity has a pull near zero - its repair waits, however high-priority the bread it could make, because spare ovens already suffice. An oven that is the binding bottleneck for heavily-desired bread pulls hard; one that serves only a lightly-weighted cake pulls weakly even when fully strained. And the pull is **capped from above** by the next-best path: if an alternative recipe

[4]Formally this pull is the gradient of the oven's possibility-tension - the dual variable of its capacity barrier, the object Cockshott computes as the labour-valuation of a producer good under linear-programming planning. We drop the term "shadow price": **price** because there is none (no exchange, nothing a commodity carries) and **shadow** because nothing is imputed - the pull is computed in the open, the federation valuing its own bottlenecks to itself under governance. A marginal labour-equivalent, not a value the good possesses.

can bake without that oven, the oven's pull can never exceed how much worse the alternative is. A rule that merely passed the highest downstream priority up the recipe graph would over-invest in slack capacity; the pull holds back.

The plan itself *is* solved - one global, certified convex optimisation, and a cheap one (§One program). The federation re-runs that solve each plan; **between** plans, harmony is **metabolised** in real time. So the pull is the **ideal** the solve computes, and the cascade is its moment-to-moment **realisation**. When heavily-desired bread is starved because the oven is down, bread's desire-tension stays open, and because the deficit is quadratic it **grows loud**. That loudness propagates upstream through the recipe graph: the humble oven-repair inherits the loudness of the bread waiting behind it. The propagating loudness is the distributed realisation of the pull - bottlenecks shout in proportion to the desire they hold up, and labour flows toward the loudest. The curvature of harmony sets the volume.

Two consequences follow. A bottleneck that cannot be cleared quickly - a long replenishment lead time - pulls high **and persistently**; unable to relieve it by repair, harmony escalates to a reserve-draw or an alternative recipe. This is exactly where derived demand meets the possibility-tensions below: a persistent, hard-pulling bottleneck nearing its limit **is** a fragility the federation must see coming. And a means of production shared across many final goods needs no rule to apportion its urgency: its scarce capacity goes to the highest-desire use at the margin, and the pull is simply the value **there** - contention resolved by the same harmony that resolves everything else.

2.4.2. The floor

A desire-tension is **bounded**: a wholly unmet want costs its weight w_d and no more, so a want priced far above the rest can be set aside for a finite price. What must never be traded away is therefore written as the other kind of tension. A **reproduction floor** is a possibility-tension: a barrier, divergent as its slack closes, which no weight on any desire can outbid. Beneath the floors the plan is a feasibility; above them it is an optimisation over desires. What a federation decides is where the floor lies - which tensions are possibilities and which are desires. That decision is political, and the arithmetic is deliberately unable to make it. The measurement behind this - how closely harmony tracks equal satisfaction above the floors, and how a want and a floor differ beneath them - is in the appendix *Why the objective can be trusted*; the full catalogue of floors and their fragility profiles is in *The living plan*.

This is the shape of **every** possibility-tension, and the ecological case shows it most clearly: the canonical barrier, $\varphi(s) = -w \log s$, that *Possibility-tensions and their fragility profiles* below reuses for every limit the federation tracks. A natural agent does not fail at a single threshold - it degrades gracefully across a range, then collapses qualitatively past a **tipping point**: a fishery whose breeding population drops too low, a soil microbiome stressed past recovery, an atmospheric concentration past which feedbacks self-amplify. So the federation carries a **profile**: a curve mapping the agent's state s to its possibility-tension - negligible near the healthy steady-state $s_\star$, rising non-linearly as s approaches the tipping point s_{tip}, diverging at the tipping point itself. The natural form is a logarithmic barrier:

$$\varphi(s) = -w \log s, \quad s = \frac{s_{\text{raw}} - s_{\text{tip}}}{s_\star - s_{\text{tip}}}$$

Far from tipping, the tension is negligible and its **pull** near zero - harmony allocates freely. As state degrades, the pull rises continuously, until pressing further would cost more labour than the desire

it satisfies, and the federation eases off **by its own arithmetic** before the limit. At the tipping point the tension diverges and no allocation can include the draw. This steepening-toward-divergence is **fragility**: the hard cap is simply the limit a possibility-tension approaches - felt continuously, in the same labour-hours as everything else.

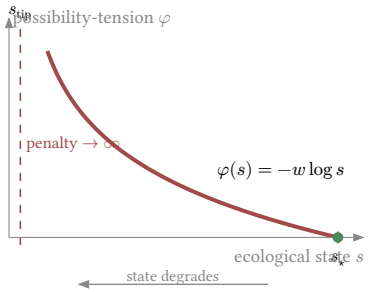

Figure 2: **A possibility-tension.** A logarithmic barrier maps an ecological agent's state s to its possibility-tension in labour-hours. Near the healthy steady-state $s_\star$ (right) the tension is negligible and its **pull** near zero, so harmony allocates freely; as the state degrades toward the tipping point s_{tip} (left) the pull rises continuously, until pressing further costs more labour than the desire it satisfies - the federation eases off by its own arithmetic before the limit. At s_{tip} the tension diverges and no allocation can include the draw: this steepening-toward-divergence is **fragility**, and the hard cap is simply the limit it approaches.

2.4.3. Reproduction labour and the two measures of labour

The **crystallized** measure of *Vocabulary* is found by a **value rollup** over the recipe: the direct labour at this stage plus the crystallized labour of every input **consumed** - each weighted by the quantity used and carrying its **own** $\text{SNLT}_{\text{crystallized}}$ - and the depreciation of every means of production **used** - the wear δ it sheds per use times **that means'** $\text{SNLT}_{\text{crystallized}}$. Depreciation is the **sole** channel by which a means of production enters the goods it makes: **repair** is direct labour **on the means itself** - building and repairing both raise the means' condition, hence its $\text{SNLT}_{\text{crystallized}}$, which then transfers into the product only as the means wears.[5]

A skill enters no good this way. **The skill is not a means of production; it is the other half of the forces the means serve** - so its own $\text{SNLT}_{\text{crystallized}}$, the training that produced it, is booked once, as its own reproduction cost (*The categories of social reproduction* below), and never rides into the goods a trained hand later makes the way a tool's build cost rides into goods it touches, via wear. And within $\text{SNLT}_{\text{crystallized}}$ every skill's hour enters at **coefficient one** - a surgeon's and a baker's alike, kept apart by kind, never weighted. A skill's **scarcity** is a different object: a pull λ_k on its availability, the same species as an ecological rent, which acts on what the plan allocates

[5]Marx treats repair two ways in *Capital* Vol. II, Ch. 8, and the two ledgers hold both. **As living labour** it is a running expense - "labour spent on [the machine], in which it is not an agent of production but raw material," classed with circulating capital, "sui generis… among the running expenses"; this is the repair's own direct-labour cell on the surface, counted once in L. **As value** it is renewal: it keeps the means in condition, and "the average life itself is based on the assumption that the additional capital required for maintenance is continually advanced" - so the repair labour is carried into goods through the means' depreciation, not as a separate term in their crystallized cost ("repairing means renewing… a constant state of rejuvenescence"; in practice repairs are even booked into the depreciation rate, which "does not change the actual state of affairs"). So the same wearing splits cleanly (developed in *Reproduction labour: deductions, target, and inspection* below) into **maintenance** - the repair, restoring what the use can recover - and **depreciation** - the value transferred as the means is used up - without double-counting: the living hours land once in L, their value rides into goods once via depreciation.

and on the training it calls for (*Production, distribution, exchange, consumption*) and never on what a good costs to claim.

And that value is reckoned at the means' **cheapest current reproduction** - built afresh or repaired, whichever is socially necessary - so maintaining a durable, when repair is the cheaper way to keep it in condition, lowers the depreciation each product carries: the same value spread over a longer life. The longer life and the lower per-unit charge are one fact.[6] This propagates recursively from raw materials to finished good. A recipe that yields two products at once - wool and mutton from one shearing - has **one** labour cost, and the rollup splits it across the co-products in proportion to quantity times what any other recipe charges for each, a co-product only this recipe reaches taking its siblings' mean; the shares sum to the whole, so no by-product rides free and no hour is counted twice. The plan's reported $\mathrm{SNLT}_{\mathrm{crystallized}}$ and the price a claim pays are one number under one rule. A co-product's **value** at the margin is the plan's own dual for it, read off the solve (*Concrete and abstract labour*) - the labour content is what that dual reduces to when nothing but labour is scarce, and it never reads the dual back.[7] A resource that **contains** others carries their value too: the embodied total of a loaded container is its **own** assembling and handling labour plus the $\mathrm{SNLT}_{\mathrm{crystallized}}$ of each resource **contained** in it - separate and additive, never folded into the container's own figure - so labour-content rides with the contents through every box, transfer, and transformation. The containment cascade events each draw-down as the container is drawn on, so nothing is lost when a thing is boxed and the box becomes something else.[8]

The two do different work. **Direct labour** is what the surface renders: each cell's area is the living labour spent reproducing that resource this period, so the areas sum - each hour counted once, where it was performed - to total social labour L. This is labour appearing directly as a component part of total labour, not as a value the product possesses.[9] $\mathrm{SNLT}_{\mathrm{crystallized}}$ is the **derived** measure: it sizes the work-intents in the plan and settles the labour-certificate drawdown - what a claim costs society - transparent bookkeeping for distribution, never an exchange-value. Each may carry distinct values for maintenance, expansion, and insurance. Labour is allocated through the concrete plan, and the matching that fills it.

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[6] This is **moral depreciation** (*Capital* Vol. I, Ch. 15): a means of production loses value not only by physical wear but by any cheaper way of reproducing it - so the value it transfers is its **current** reproduction cost, not its historical outlay. Because that figure is one socially-necessary number for the means (computed by the value rollup over the whole recipe graph), a single improvement in keeping a means in condition lowers the $\mathrm{SNLT}_{\mathrm{crystallized}}$ of **every** good that wears it at once - each in proportion to how hard it wears it - and so the credit cost of claiming each of them falls together. The reduction is bounded by how much of the wear repair can actually recover; the irreparable remainder is still made good by replacement, at the means' build cost.

[7] Marx, *Capital* Vol. I, Ch. 1: commodities "are but crystals of this social substance, common to them all - they are Values." The distinction between **living** labour, in the act, and **dead** or **crystallized** labour, congealed in the product and carried forward by the means of production, runs throughout *Capital*. The point here: **both** the living labour added now and the crystallized total are **socially necessary** - so both are socially necessary labour times, marked **direct** or **crystallized**. Each is always written with its qualifier: the surface uses the direct measure $\mathrm{SNLT}_{\mathrm{direct}(r)}$, while credit and sizing use the crystallized measure $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$.

[8] This is what keeps the value rollup whole across containment boundaries. When a process consumes a container, the cascade emits a consume event for each thing inside it, so the contents enter that process's "inputs consumed" term at their own $\mathrm{SNLT}_{\mathrm{crystallized}}$ - and the container's own figure counts only its assembly, never its contents, so the two never double-count.

[9] The surface measures living labour directly, never an embodied value. The old question of whether the proceeds of labour are the total value of the product or only the value labour newly added names exactly these two measures: $\mathrm{SNLT}_{\mathrm{crystallized}}$ is the total, $\mathrm{SNLT}_{\mathrm{direct}(r)}$ the newly-added, and it is the newly-added that the surface shows.

2.5. The surface

The federation's plan is rendered as a treemap[10] - shown in Figure 1 at federation scale and Figure 2 at scope scale - and we refer to this rendering throughout as **the surface**, and to each rectangle on it as a **surface cell**. The surface is the labour-time ledger of **social reproduction**: not the goods the federation holds, but the living labour it spends each period to keep its material condition at the level it wants. Each cell's **area** is its reproduction labour - the living labour spent this period to (re)produce one resource **at its own stage** - computed as the quantity reproduced times its direct labour per unit. The embodied labour of inputs is counted in the inputs' own cells. A cell's **tab** names the target: the standing level the federation declares it wants on hand. The two-tone **fill** separates realized labour (dark) from labour still to come (light).

$$\underbrace{\text{area}(r)}_{\text{cell area}} = \underbrace{R_r}_{\text{reproduction labour}} = \underbrace{q_r}_{\text{quantity reproduced this period}} \cdot \underbrace{\text{SNLT}_{\text{direct}(r)}}_{\text{direct labour per unit}}$$

The surface sums to $L = \sum_r q_r \cdot \text{SNLT}_{\text{direct}(r)}$.

The apportionment of L (total labour) across scopes - how much social labour flows to food production, manufacturing, education - is the **plan** rendered spatially.

[10] A treemap is the most space-efficient way to render proportional data in two dimensions: every pixel of the canvas carries information, with no whitespace or axis overhead, and the area of each cell is directly proportional to its value. Pie charts waste area on labels and arc length; bar charts waste an entire dimension. The treemap uses both dimensions for quantity.

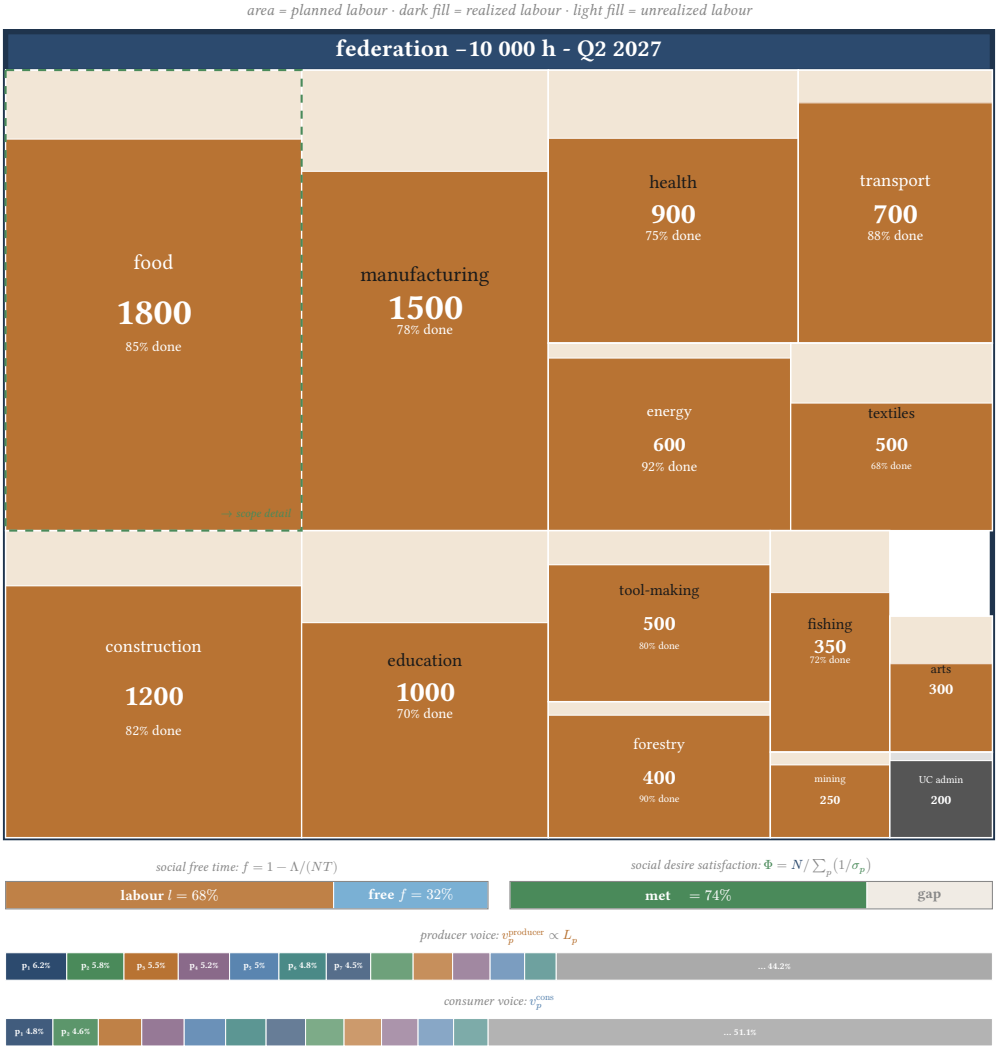

Figure 3: **Federation-level labour allocation.** Each cell is a scope (branch of activity); area is the scope's **planned labour** ($q \times \ell$, direct labour, summed across its leaves). The two-tone fill: dark amber from the bottom is **realized labour** (matched hours already worked); the lighter zone above is **unrealized labour** (matched hours still in the future). The grey UC admin cell is federation-level administration - verification, plan coordination, refederation - matched from the federation pool. Two voice bars: **producer voice** ($v_p^{\text{producer}} \propto L_p$ - shapes how the work is done, more unequal) and **consumer voice** (v_p^{cons} - shapes what the community needs, more equal). Two measures: the social free time $f = 1 - \Lambda/(NT)$, with Λ the hours the plan places, shows the fraction of social time that is free; the social desire satisfaction Φ shows the harmonic mean of individual satisfaction - structurally sensitive to whoever is worst off. The social play time $\mathcal{P} = \min(f, \Phi)$ measures the joint limit: $\mathcal{P} \to 1$ only when both free time and equal social desire satisfaction approach 1.

The total product of the community is a social product, and the surface renders the labour of its **reproduction** - not only producing anew but maintaining what exists, expanding it, and insuring it against calamity, all it takes to sustain the community's material condition. Every matched hour appears as area in a single readable surface, denominated in labour-hours from the federation down to the individual claim. Zooming into a single scope (Figure 2) reveals four categories: **means of production** (remains social), **skills** ((re)production of labour-power), **means of consumption** - distributing through two rails: **individual claimable** (redeemed against personal credit) and **communal** (socially regulated use) - and **administration** (the costs of governance, minimised by design). Within each scope, **producer voice** (a citizen's share of the scope's labour) shapes **how** the work is done; **consumer voice** (*The credit identity*) shapes **what** the community needs.

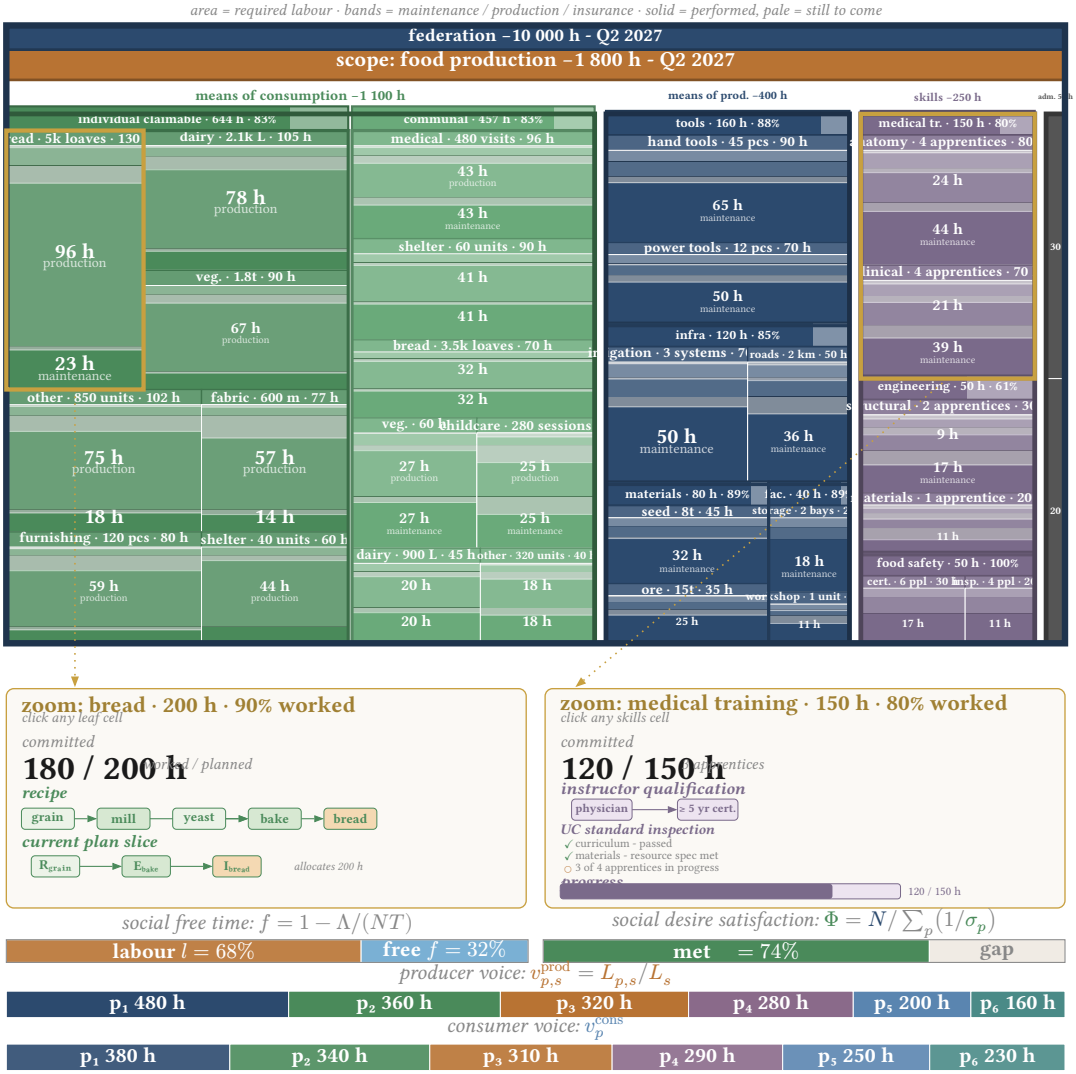

Figure 4: **Scope-level plan with nested categories.** Area is proportional to social labour-hours allocated. Four categories partition the scope's 1 800 h: means of consumption (green, 1 100 h), means of production (blue, 400 h), skills (purple, 250 h - stock of skilled people, the (re)production of labour-power), and administration (grey, 50 h - scope-level self-governance). Within MoC, two distribution modes: individual claimable (dark green - redeemable against personal credit) and communal (medium green - distributed by social decision). VF resource classifications appear as header bands; leaves are the **economic resources themselves** - the actual stock of things the federation reproduces (the real loaves, the real housing units, the real trained people), grouped by their resource specification. Each leaf cell's tab carries its target and the required labour; the cell body decomposes that labour by **intent** into the three deductions - **maintenance** (the solid base), **production** (expanded reproduction, lighter), and **insurance** (the reserve, lightest) - sized by share of the total. Within each band the labour already **performed** is the vivid solid; the labour still **to come** is a paler, desaturated cap, so every intent shows its own progress. Consumables read as mostly production (made fresh each period); durable tools and infrastructure as mostly maintenance. Zoom panels: left - bread recipe DAG (grain → mill → bake → bread) and current plan slice; right - medical training with instructor qualifications and UC inspection checklist. The producer voice bar shows $v_{p,s} = L_{p,s}/L_s$; the consumer voice bar shows v^{cons}_p.

Every cell in Figure 2 carries **two readings**. The **area** is denominated in labour-hours - the reproduction labour the plan allocates this period, computed as $q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}$ (the direct labour added at this stage). The cell carries its **target stock** in natural units: 200 housing units, 50 surgeons, 5 000 loaves this quarter. The target is the standing level the federation aims to hold on hand, and the distance between target and what is actually on hand is what summons the period's reproduction labour - decomposed, where the target persists, into **maintenance**, **production** (expanded reproduction), and **insurance**, each its own recipe and direct labour. The surface renders that labour as area, and the cell body splits it into those three **intent** bands - lightness naming the intent (maintenance darkest, the reserve lightest), each band sized by its share of the total - while within every band the labour already **performed** is the vivid solid and the labour **still to come** a paler, desaturated cap. A citizen reading the surface sees simultaneously **what level** the community aims to sustain, **what living labour** it costs, **to what end** that labour is spent, and **how much of each** has been realized.

2.6. The categories of social reproduction

The plan allocates social labour **proportionally** across categories of what is reproduced, each an economic necessity whose magnitude is determined by available means and forces and by the computation of probabilities. Four are reproduced **for use** - drawn into production and consumption; alongside them the federation reproduces the **ecological base** itself - the ground those four draw on - held apart as its own reserve. The four for use are:

Means of production - the blue section of Figure 2. A **stock** of tools, infrastructure, materials, and facilities the federation holds on hand, whose level bounds the labour every recipe can draw through them. A means is **used**, not consumed: it does its work and **remains**, whole, for the next hand - so what the plan rations is not its substance but its **time**, its capacity occupied for a period and handed back. Maintaining an existing oven, enlarging the oven stock toward a higher target, and holding its spare-parts reserve - the three components in concrete form - are distinct leaf cells under the same classification, each its own recipe and crystallized SNLT. Reproduction is pulled by actual drawdown.

Skills - the purple section. The federation's **stock of skilled people** - the (re)production of labour-power, rendered as the catalogue of skills the federation sustains. For each skill k in the catalogue $\mathcal{K}$, the plan carries a target quantity in the natural unit of this category: **number of people trained in *k*** - 20 surgeons, 50 bakers, 10 welders, 8 structural engineers. Each cell's area is $q_k \cdot \mathrm{SNLT}_{\mathrm{direct}(k)}$ where q_k is the targeted number of people and $\mathrm{SNLT}_{\mathrm{direct}(k)}$ is the **direct** labour spent forming and sustaining one such person this period; the full embodied cost $\mathrm{SNLT}_{\mathrm{crystallized}(k)}$ - rolling in the materials and facilities training consumes - is the separate measure used for sizing and credit. Skills is the federation's **capacity stock of skilled population**: like means of production, it is a stock held on hand, and its level bounds the labour every recipe requiring skill k can draw. The same maintenance / expansion / insurance decomposition applies, now with people as the unit being sustained:

Skill formation - training new people in a skill: **maintenance** when it replaces those who leave, retire, or age out, **expansion** when it grows the trained count beyond its current level. Apprenticeship, formal study, certification. Skills enter the catalogue $\mathcal{K}$ by two paths, distinguished by **irreversibility of error**. Most skills are generated inside the production process itself - a byproduct of doing the work, where mistakes are reversible. Other skills

(surgery, structural engineering, vaccine preparation) require study to **precede** production because a failed attempt causes irreversible harm.[11]

Skill maintenance - sustaining existing trained people. A surgeon who does not operate degrades; a language not spoken atrophies. Most upkeep rides **free on the work itself** - exercising a skill in production keeps it sharp, the same hours seen twice - so maintenance is a by-product wherever demand draws on the skill. Only an idle stretch, with no work to practise against, needs its own refresher: recertification and continuing practice as a distinct activity with its own crystallized SNLT - the labour to keep a living skill alive is not the same as the labour to form it initially, and a skill kept warm **by doing** costs nothing beyond the doing.

Skill redundancy - additional people trained in the same skill, so the federation is not fragile if any one of them leaves, falls ill, or ages out. This is the **insurance** component in human form, but a **living** reserve, not an idle one: a skill unpractised decays, so the redundant practitioners stay active and current. It is a **target** decision sizing the margin of trained people sustained beyond bare sufficiency - the measure of how far the federation keeps each skill from its fragility threshold, sized, as Marx specifies, by the computation of probabilities rather than by equity. The same margin is the **skill redundancy** profile of §Possibility-tensions and their fragility profiles - the one decision, read there as tension.

Trained people appear in two places on the surface. The **stock** of trained people lives in this purple section, indexed by skill, quantified in persons. The **exercise** of those skills - a baker baking bread, a surgeon performing surgery - appears as work flows in recipes throughout the green, blue, ecological, and admin sections. One sustains the capacity; the other applies it. The relationship is identical to means of production: an oven is a stock in blue, and the oven's use is a flow scattered across every recipe that needs baking.

The purple section's total area is the **direct labour the federation spends (re)producing social labour itself** - the hours it invests to sustain its skill base at the levels the plan requires. Each skill formation, each recertification, each cross-training slot has its own recipe and direct labour; the embodied cost $\text{SNLT}_{\text{crystallized}(k)}$, into which the value rollup folds the labour of an instructor, the materials consumed in training, and the facilities used, propagates through the recipe graph for sizing and credit. As productive forces develop and crystallized SNLT falls across the economy, the skill composition shifts - less routine labour, more monitoring and regulation - and the (re)production cost of the new skill mix may rise even as total labour falls. The federation makes this visible on the surface and governs it through the same planning mechanism that governs every other category.

Means of consumption - the green section. The share of the social product destined for personal and communal use. It holds both **stocks** and **services**. Durable goods (housing, clothing, furniture) are stocks held on hand, their reproduction labour carrying maintenance, expansion, and insurance as separate leaf cells. Consumables (food, medicine) are stocks too, held as thin buffers between steady output and consumption, where maintenance is simply the continual re-making that replaces what is drawn. **Services** - a consultation, a lesson, an hour of care -

[11] The distinction shifts as productive forces develop. A skill that once required formal training (typesetting, navigation) may become safe to learn on the job as tools improve and error consequences shrink. The skill catalogue tracks which skills currently require prior certification and which can be acquired through production.

hold nothing on hand: they are reproduction labour consumed in the moment it is produced, bounded by the skill and facility stocks they draw on.

How a product reaches people is a separate axis from **what** it is. Both goods and services flow along the same two rails:

> **Individual claimable** - drawn against the citizen's consumer voice v_p^{cons}. Each citizen's claim capacity is $W \cdot v_p^{\text{cons}}$, where W is the pool's labour-content together with what has been claimed from it, and their drawable credit is that less their drawdown; they draw according to their priority weights. A loaf claimed as your own, a consultation booked as your appointment - the rail is the same, and the rail is a recipe: a **claim** consumes the scope's good and the citizen's credit at the good's $\text{SNLT}_{\text{crystallized}}$ and produces the citizen's own (`bread~ada`), which the **eat** recipe then reduces to the nutrients a floor is stated over - custody moving because the flow says so, the way a transport recipe moves place. Credit is drawn and tallied there, never a wall: a reproduction floor stays a need whatever a citizen holds, and the certificate's balance is checked where the claim becomes an event - a planned claim is a commitment, and the citizen's confirmation records the transfer against it, the way a confirmed shift records its work.
>
> **Communal** - allocated by social decision and consumed in common: public shelter, a vaccination campaign, schooling, a public lecture. Distributed according to the aggregate social desire, weighted by consumer voice.

So the same consultation-labour is claimable when it is your private appointment and communal when it is a public-health programme: the stock-or-service question and the claimable-or-communal question are independent, and every cell answers both. Which rail a good is on, the verbs say: what a process `uses` is a **means**, what a scope's own recipe consumes is **communal**, and what a claim recipe consumes for someone - or what is made or stocked and is nobody's input - is **individual**. Two rules follow, and everything else reads them. What a process uses, no person claims: an oven or a field gets no claim edge however it is asked for or tagged, its custody the custodian-scope projection, its title the ownership credential's. What a person holds, no plan takes: the loaf a claim handed a citizen enters the next solve as their own, `spec~aid`, met from before anything is claimed; a means they hold waits for the lend. The plan publishes the rail once per solve; the pool, every claim, the custody invariant and the legal channel read that one word, and a record's own vocabulary decides nothing.

Administration - the labour of answering: does the intent match the specification? This applies universally: work intents against their specifications (were the hours real, do the skills correspond?), intents to produce or consume against the resource specification (does the output meet the spec?), and **ecological specifications** against the regenerative capacity of natural systems (does the draw respect the envelope?). The question is the same whether the specification describes a loaf of bread, a surgical procedure, or an ocean fishery. Two levels:

> **Scope-level self-governance** - the internal answer. Plan formation, coordination, standard compliance, handling cases of visible non-contribution. Every scope that touches natural systems carries ecological specifications in its plan - the farmer verifies the soil, the fisher verifies the catch, as part of the same self-governance that verifies any other specification. The people who verify the work **are** the people who do the work.
>
> **UC-level verification** - the independent answer. The scope records its own economic events; the UC verifies them against observable reality. Do the 5 000 recorded loaves exist?

Does the inventory match the ledger? Does the output meet the resource specification? Were the hours real? Does the fishery draw stay within the regenerative envelope? Verification is independent of the scope being verified - the UC draws verifiers from the federation pool by the egalitarian matching principle. Results feed forward into future plans, adjusting recipes, skill requirements, allocations, and targets.

Administration is itself productive activity: its processes have their own recipes, and the concrete needs of production determine what administration looks like, how often its actions occur, and what form they take. The design goal is to minimise this category. These hours appear on the surface as area - visible, accounted for, and targeted for reduction as the federation develops.

Within these four, the three intents recur (maintenance, expansion, insurance - developed in *Reproduction labour: deductions, target, and inspection* below - and inspection, verifying all three, developed in *The surface as a stock network*). **Insurance** is the one to weigh here: it is the **reserve component within each of the four** - a spare oven, a redundant practitioner, a reserved stock - the **hard**, probability-sized member of the three, summable across the surface into the reserve fund.

One reserve has no use-good to belong to - the **ecological base** itself, the ground all four draw on:

Reserve - the ecological base - what the federation reproduces of nature's own renewal: forests (a standing aim of, say, 50,000 trees), fisheries, soils, biodiversity, the atmosphere. Nature holds no cell among the means of production or consumption - it is what they draw upon - so its reproduction stands as its own category, held apart from use. Reproducing it is the federation's conscious **regulation of its material metabolism with nature**: keeping the natural base whole. The living labour is largely that of watchman and regulator - nature, once understood, doing its own reproducing.

Across every category the surface reads the same way - area is reproduction labour, the cell's label its target stock, the two-tone fill realized labour below labour still to come, and **your labour** gold within individual-claimable cells. It is a single legible dashboard of the federation's **reproduction**: the total living labour required to sustain its material condition, allocated across capacity, consumption, administration, and reserve, rendered spatially.

2.7. Governance

Every governance act falls into one of two classes:

Production governance - how the work is done. Recipe selection, plan verification, scope self-governance, capacity-recognition, scope formation. Weight is producer voice alone, read at the grain of the **part** the act concerns: a citizen's share of the validated hours worked on that part. The scope is the standing part, and there the weight is the citizen's share of the scope's total labour. Those who do the work shape how it is done, and they shape each piece of it in proportion to how much of that piece they do (*The voice field*, below).

Consumption governance - what the community needs. Citizens compose the social desire through priority weights spanning both the individual-claimable and communal rails of the means of consumption. Weight is consumer voice, defined in *The credit identity*. The three components - contribution (proportional to your labour), solidarity (proportional to your reduced capacity), and citizenship (an equal share for every citizen) - are the same equation read as governance weight. Everyone - workers, children, the elderly, the unable - carries weight in shaping what the federation produces for collective and individual use.

An energy scope's producers shape how energy is produced; the federation shapes whether to produce nuclear energy at all. Matched hours do two jobs in a single act: they determine your producer voice in shaping what the scope produces, and they grow your consumer voice - determining how much of the individually-claimable pool you can draw. One number, two functions, assigned in the same moment. As social free time rises - productivity grows, less total labour is needed - both voice and credit converge toward an equal share for every citizen. The labour-measure recedes because **the social-material ground that required rationing has faded into abundance**.[12]

2.7.1. The voice field: how the work is done

A plan has thousands of pieces, and a yes or a no on the whole says nothing about which piece. Production governance is therefore exercised **on the part**: a person points at a piece of the plan and takes a stance on it, the stances of everyone who works that piece are weighed by their hours on it, and the solve feels the sum as a bounded re-weighting of what the piece costs. Nobody votes on the plan; the plan is solved from the stances, and the scope verifies it.

A part is a conjunction of nodes drawn from the classifications the community shares - the scope tree, the process and its recipes, a means at the grain of its specification, a place, a window from a small governed set (nights, early, weekend), a skill, and a folksonomy tag naming a family the structural kinds do not carve out on their own (a chain of recipes joined by real flow: "the co-op-mill, wood-oven, cargo-bike way of making bread"). At most one node per kind, all holding at once: "nights, at this bakery, on this kind of oven" is one part. A part may narrow and may never join. A union has no name under this grammar, so nobody can stitch their own work to work they never touch and carry their hours there; on the half they never worked, their share is zero. A part compiles, at each assembly, to the set of columns it covers, and a part that covers nothing this assembly is said and bends nothing. A person never composes a conjunction: they point at something in their own calendar, and the face offers the rungs of one ladder - just this shift, the early shifts here, nights in general, this way of making bread - each rung the instance's own terms with some released.

A stance is a number in $[-1, +1]$ on a part - dislike to like - and it is a statement about the plan: **less of this, or more, for anyone**. Its weight is the speaker's producer voice at the grain of the part: their share of the validated hours worked on it, read fresh from the work log through the same gate credit uses, so the two can never diverge. A person's stances form a **hand** held hour by hour: over the parts that any one of their validated hours falls in, their portions sum to at most one, and hours that share no part speak separately. Each hour speaks at most once, so a broad stance spends every hour it covers, meaning everything loudly means nothing, and **rotation costs no voice**: a person who works six parts speaks on each with a full hand and their share of it, and their say follows their hours wherever those fall - $v_{p,s} = L_{p,s}/L_s$ at every grain. A person who has never worked a part moves nothing on it by this door; the nodes above it - its process, its scope - carry the say of those who do that kind of work. Voice is radically local at every size, and the scope's

[12] Marx, *Grundrisse* (Notebook VII, "Fragment on Machines"): "The development of the social individual appears as the great foundation-stone of production and of wealth. As soon as labour in the direct form has ceased to be the great well-spring of wealth, labour time ceases and must cease to be its measure. The free development of individualities, and hence not the reduction of necessary labour time so as to posit surplus labour, but rather the general reduction of the necessary labour of society to a minimum, which then corresponds to the artistic, scientific etc. development of the individuals in the time set free, and with the means created, for all of them."

fold publishes hours per cell of work and mass per part with no person in any cell, so nobody is visible above their own scope.

The field. Every column j already carries a weight in the objective - its hours, its draw on stocks at crystallized SNLT, its acquisition cost, all positive, all in hours. The pushes of the parts covering j add into P_j, each part's push the voice-weighted sum of its stances, and the column's stance is what is left of that push once every statement about goods is taken out:

$$s_j = \operatorname{clamp}\left(P_j - \sum_g \operatorname{into}(j,g)\cdot\lambda_g, -1, 1\right), \qquad w'_j = w_j\cdot\left(1-\tau\cdot s_j\right)$$

with $\operatorname{into}(j,g)$ the share of column j's hours that go into good g (the same attribution the value rollup makes of crystallized hours), λ the numbers that leave each good's felt hours equal to its clock hours at the worked mix - one small symmetric system, goods by goods, solved before the solve - and τ a governed bound strictly below one. Three things hold by construction. **A push shared by every way of a good cancels**: "less baking" laid over every way bread is made is a statement about **what**, and its home is consumer voice. **A good with one way of being made has nowhere to move**; its cost at the worked mix stands, and the stance waits for a second way to be mapped. **At the mix actually worked, felt hours equal clock hours**, so the field leaves the cost of every good where the solver put it and moves only the choice among its ways; how much is made stays with the wants. And the form rules out what it must: no column's weight reaches zero, so a liked way becomes cheaper and never free and voice cannot mint a direction the objective does not feel; the field re-weighs cost and holds no desire of its own; and it is soft everywhere - floors, ecological walls, element conservation, decided columns and the legal gate stand exactly as they did, and the standing plan is feasible whenever the potential plan is. Between two ways of equal cost the liked one runs; against a way far cheaper, a dislike of bounded weight loses, and the report says by how much.

What voice displaces is read by the one difference: the standing program against the same program at zero field, at one settled floor - hours added, draws added on each wall, met need lost or gained. A part's displacement at the margin is $-\tau\sum_{j\in\text{ part}}\ell_j x_j$, exact by the envelope and additive across parts; a saturated column reads zero. It is shown to the people holding the stance and to everyone the cost falls on. A dislike the scope keeps paying for is the signal to hold a ballot: **a ballot makes a wall** ("never this recipe here" lands as a forbidden column), **an accept decides a column** (a person's own promise fixes it), and stances fill the space between - everything a community means **other things equal**. The field is where a prohibition is rehearsed before it is enacted.

What acceptance becomes. Each judgment a plan vote carried lands on the instrument that fits it: whether the work is worth doing is the wants that pull it (unwanted work is never scheduled); whether the recipe is sound is its map, at a confidence, audited; how it is done is the field; never-this is a wall by ballot; whether **I** do it is each person's own accept; whether it was done, for someone, is the recipient's confirmation; a split or a merge is a ballot. The plan that stands is a certified function of governed inputs, and what a scope does with it is **verify** - the certificate and the identifiers of the inputs it was solved from. An hour is social labour when it fulfils a commitment the plan offered and the person accepted, times the recipient's satisfied fraction, and it reaches as far as the solve that placed it. No later act erases hours worked under a certified plan and an accepted offer: a change of voice bends the next plan, and a false map is a finding against the statement, never against the worker who relied on it.

The two rails. A person at a shift can mean two things, and each has its rail. **"Less of this, for anyone"** is the stance above, on the **how** rail, weighed by hours. **"Not for me"** is a statement about the person, on the **who** rail of the egalitarian matching principle - willingness, the personal accept, a preference over one's own slots - where every citizen's say is equal whatever their hours, and it spends no hand, since it touches only that person's own assignment. The field carries the first statement alone: a column's weight is one number for whoever works it, and a private dislike sent through it would move work away from people who want it. Where one baker finds nights hard and another likes them, the answer is a matching, at no cost to anyone, and the who rail finds it. A private preference is read **equal and bounded**: clamped to a governed range $[-\beta, \beta]$, centred on the person's own mean over the skills they hold in the plan, in the congestion term's own unit, so two citizens stating the same preference move their own assignments alike whatever their hours, equal reluctance all round leaves every assignment where it was, and a single-skilled person's preference says nothing about how much they work. Hours buy voice and never assignment. Refusals still reach the plan, as facts: few willing hours on a cell make that labour scarce there, and the solve prices the scarcity with nobody having claimed the part is bad.

Feasibility follows the map. "This won't work" is a claim about what **is**, and it has a different home from a claim about what is **liked**. The field can make a way dearer by a bounded fraction; a wrong figure needs to leave the feasible set, and no stance can take it there. So it goes through the map: a **contrary observation** names a mapped figure of a part - a yield, an input rate, the hours a step takes - states a reading witnessed by a work event of the observer's own, and is signed by them. It is a constative act, guarded the way maps are: by who may speak (a worker of the part, their witness event on record), by one live observation per observer, part and action, and by audit. It moves the figure in the cautious direction alone - a produce read at most the nominal, an input or the hours at least the nominal - laid over every scenario the robust solve runs, and it touches nothing else. Two workers observing the same failure are two witnesses; a hundred who never touched the figure are none. The nominal figure stands, so a corrected recipe restores it by the ordinary path.[13]

[13]Realized as a field behind one governed number, `voice-tau`, which defaults to zero, so every solve reads the plan as if no stance stood until a scope sets it. `module/playnet/planner/facts/parts.scm` is the selector (one optional slot per kind; a part that covers nothing is the symbol `nothing`); `engine.scm` `key-terms` / `column-terms` is the one reader of what a column is made of, and `observer.scm` `hours-on` the one read of validated hours on a part (`voice-share` is it on the scope, the widest part; `hours-by` the fold beneath it); `planner/solve/voice-field.scm` holds the arithmetic pure - hand, shares, pushes, the reference by the normal equations, the clamp, `field-delta` and `voice-marginal`; `pools/voice-fold.scm` is the scope fold, seeded from the standing hands when the scope's folds are wired; `governance/service.scm` `cast-part-signed` lands a signed hand as an expression-stance credential of domain `plan-part`, the newest per citizen standing; `solve/externality.scm` `paired-difference` reads the price - the one difference operator regret and the three bills are written through too - and the planner's `voice-displacement` door the total beside the marginal. The who rail's bound is `*pref-bound*` at the one site the preference is read, governed with τ_v, ρ and κ from the one registry in `governance-policy.scm`; the contrary observation is `playnet-contrary-observation/v1` in `governance/credentials.scm`, folded by `engine/robust.scm` `observed-shocks` through the same shock vocabulary a realized shortfall already calibrates. Proved in `docs/formal/solver-certificate/SolverCertificate/{Parts,VoiceField}.lean`; run as `tests/planner/test-property-voice-field.scm` (the bound, the hand, rotation), `test-voice-field-solve.scm` ($\tau = 0$ is byte-identical; the liked way runs; a far cheaper disliked way still runs), `tests/governance/test-part-stance.scm`, `test-emp-modes.scm` (the bound) and `test-contrary-observation.scm` (a half-yield oven doubles the bake hours; a flattering reading changes nothing). Left for later: the decline record on the who rail, the ladder on the face, a fading window over the hours, closure and connectedness of the tag kind.

2.8. Reproduction labour: deductions, target, and inspection

Reproduction labour decomposes by intent into three components - exactly the three deductions from the social product - each its own recipe and crystallized SNLT. **Maintenance** holds the target steady where something **persists** - repairing what wears, replenishing a buffered stock as it is drawn - simple reproduction; a means of individual consumption consumed outright carries no maintenance, only fresh production. **Expansion** raises the target and the capacity behind it - expanded reproduction. **Insurance** is the reserve fund against accidents, dislocations, and natural calamities - sized by the **computation of probabilities**, "in no way calculable by equity," so it enters harmony as a **hard** possibility-tension rather than a soft desire (developed below). The distinction turns entirely on the target: a gap between what is on hand and the target opens only because the level fell (closing it is maintenance) or because the target rose (closing it is expansion), so the same act - baking a loaf, raising a wall - is named by which way the target moved, never double-counted.[14] The surface shows, cell by cell, how much living labour holds the present and how much builds the future; and summed across the whole surface, the three are readable as **funds** - how much of the federation's living labour replaces what is used, expands its powers, and insures it against calamity.

The reserve is the floor's own margin on the good's reproduction band: the defended point sits q_r^{res} above bare need, and the reserve is what the band keeps between the two (§Possibility-tensions and their fragility profiles); under uncertainty it is the same floor run the other way, a starting on-hand that every scenario can draw on (§Reproduction under uncertainty). One mechanism, sized by probabilities, read as a deduction here and as a wall there.

Reproduction labour carries a fourth kind, of a different sort entirely. **Inspection** is not a deduction from the product - it adds no maintenance, expansion, or reserve - but the labour of **knowing** that the other three's claims are true: that a stock said to be on hand is on hand, a skill said to be held is still sound, a means said to be in spec has not silently drifted. A possibility-tension is only defended if its slack is **known**, so verifying it is itself socially necessary labour, scheduled on the same lattice and drawn from the same SNLT budget as everything it checks. How often to inspect is set at the margin - cheap beside the breakdown it forestalls, receding as reliance falls or trust accrues - developed in full, with its optimal-intensity and recertification-interval formulas, in *The surface as a stock network* below.

A target presumes something that **persists on hand** - a stock of homes, tools, trained people, or buffered goods, drawn down by use and replenished by reproduction. Its standing level has two parts of different kind. The desired quantity q_r^* is the **soft** level we want on hand: enough to meet ongoing drawdown, plus a smoothing margin against its variability (the **buffer**, sized in *Desire on the turn*). That margin is itself **part of the good's desire**, so it is the first thing to yield the

[14] The three components are the deductions a society makes from its total product before any distribution for consumption: replacement of the means of production used up; an additional portion for expansion; and reserve or insurance funds against accidents and natural calamities, sized by the computation of probabilities rather than by fairness. Maintenance and expansion are the simple- and expanded-reproduction schemes; the reserve fund is the third. The federation sizes its insurance fund by the computation of calamity probabilities (the fragility profiles introduced below), never by fairness; the soft buffer, by contrast, by ordinary variability, and it is a wanted desire. There are in fact two such triads: these three come off the **total** product, and a second - administration, common provision (schools, health), and support for those who cannot work - comes off the consumption **remainder** before individual division. The federation keeps no separate funds for the second: administration is a labour category, common provision the communal rail, and support the citizenship and solidarity components of consumer voice - the deductions dissolved into one voice-weighted distribution, whose proportions shift as the society develops (administration shrinking, common provision growing).

moment something louder claims the labour. The **insurance reserve** q_r^{res} is the calamity stock - the third deduction, sized by the **computation of probabilities** and "in no way calculable by equity" - so it is **hard**, a possibility-tension defended behind a barrier whose steepness and size are the good's **fragility profile** (developed below). The cell shows their sum, the target stock $q_r^* + q_r^{\text{res}}$ - the full standing level - and the surface renders all of it as area. The target stock is the level the plan aims at; only the part of a persisting good's stock that stands above its social portion is distributable, and the credit pool K (*The credit identity*) is read from that part. Where nothing persists - a consultation, a lesson, an hour of care, consumed in the moment it is produced - the cell holds no level, only reproduction labour set directly by the demand it meets.

3. Desire and distribution

Where the plan comes from - desire composed into harmony - and how its product is distributed, credited, and squared with the world outside.

3.1. Desire as harmony

The priority weights of *Desire, pull, and floor* compose into the objective the planner optimises. Figure 5 shows one citizen's desire space: the same resource classification tree as production, proportioned by the citizen's weights and split across the two rails.

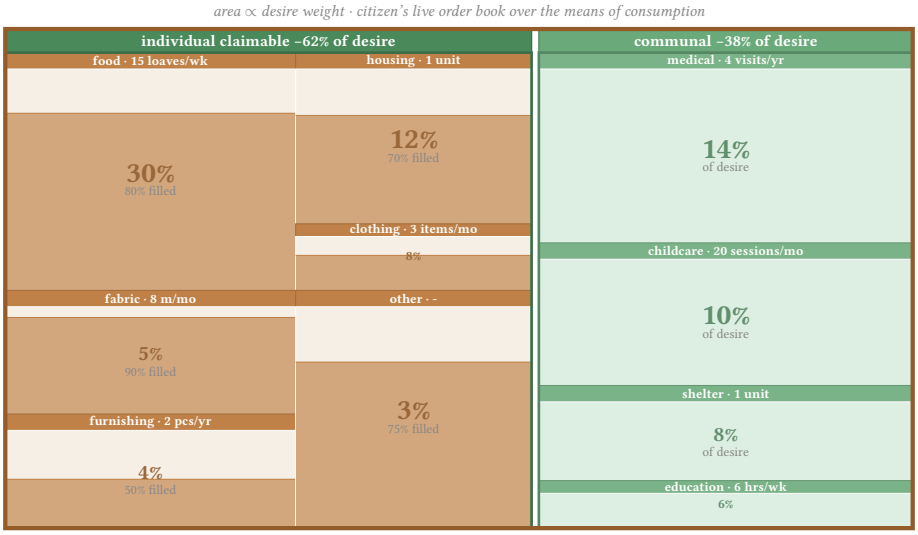

Figure 5: **A citizen's live order book in desire space.** The same resource classification hierarchy as Figure 2, rendered in desire-weighted proportions with quantities in natural units. Individual claimable cells (left) show delivery progress: the filled zone rising from the bottom is what has been delivered against the citizen's credit; the unfilled zone is what remains desired. Communal cells (right) show the citizen's weight for collective provision - delivery is determined by social decision. These proportions differ from Figure 2′s labour-hour areas; the planner generates a concrete plan to satisfy the aggregate of all citizens' order books.

[15]The term **harmony** in this sense is owed to Paul Cockshott and Allin Cottrell, *Towards a New Socialism* (1993), where it names the planner's objective function over a material plan. Their argument - a direct reply to Hayek's "knowledge problem" - is that the *informational* role of market prices is not irreducible: prices conflate three

The planner optimises the **harmony score**[15] across all demand-dimensions. This is the **desire-tension** half of the harmony field of §1: the negative sum, over every demand dimension, of the social desire weight times the shortfall squared (shortfall 0 when fully met, 1 when wholly unmet).

$$\underbrace{\text{Score}}_{\text{desire-tension, negated}} = -\sum_{d} \underbrace{w_d^{\text{desire}}}_{\text{social desire}} \cdot \underbrace{g_d^2}_{\text{gap, squared}}$$

Each term is a desire-tension $D_d = w_d^{\text{desire}} g_d^2$; full harmony subtracts the **possibility-tensions** too (the limit-barriers of §Shared budgets and possibility-tension), so the planner reads, at every margin, the **pull** of each - relaxing the loudest desire-tension until a possibility-tension pulls back as hard. The quadratic penalty **spreads partial shortfalls** across the whole means of consumption - a small shortfall on fabric to close a large shortfall on food. And because every gap is measured as a **fraction of its own goal**, equal weights yield equal fractions met: the assortment **scales together** across the scope's shared dimensions, proportional fulfilment the default between them and the weights the declared departures from it. Within a dimension, each citizen's own stated want enters through a second member of the desire family, built so that stating it honestly is each citizen's best play - *Honest by shape*, below.[16] Categories (food, housing, care) are the VF resource classifications through which the plan **produces** goods; they structure scopes and recipes. Harmony **allocates** across the entire field, reading the cross-category weights that the citizen expressed.

3.1.1. Honest by shape

A want a citizen states **for themselves** enters harmony through a second member of the desire family, the **cap**: a bounded pull equal to the citizen's weight, with the asked quantity standing beside the score as a wall rather than inside it. Because the score never reads the ask, inflating it is weakly dominated; under scarcity, service rises to each citizen's water level, so a shortage lands on the largest asks first, tilted by voice. A dimension a scope holds in common - a sub-commune's folded total, a row a group co-signs - carries the fractional-gap quadratic above instead. The proof, the family of sacrifice patterns a scope may ratify, and the audit on the one pattern the shape leaves visible are in the appendix *Why the objective can be trusted*.

3.1.2. Desire on the turn

A need is wanted in some quantity **and** at some phase of the day. Lift the gap g_d to a function on the turn $g_d(\theta)$ over a day of τ radians; the desire-tension is its mean square, which splits cleanly - by Parseval, since the turn is a circle - into a **quantity** part and a **timing** part: the squared day-mean gap, and its variance around the clock.

$$D_d = w_d^{\text{desire}} \left(\underbrace{\bar{g}_d^2}_{\text{scarcity}} + \underbrace{\text{Var}_\theta(g_d)}_{\text{cadence}} \right)$$

jobs (information aggregation, coordination of production, and profit-seeking incentive), and modern computation plus an explicit utility function plus labour-time accounting can perform the first two directly, without the third. Harmony is that utility function. Where capitalist firms read price signals to decide what to produce more or less of, the planner reads gradients of harmony - a surplus contributes diminishing returns and so signals "produce less here"; a deficit contributes a quadratic penalty and so signals "produce more here." Coordination across the economy then falls out of maximising the weighted sum, rather than emerging tacitly through exchange.

[16] The proportional-assortment guarantee is the objective of Problem C in Kantorovich's *The Best Use of Economic Resources* (Appendix I): maximise the common scale $\mu = \min_j x_j / k_j$ of a given mix. The quadratic on fractional gaps carries the same logic softly - equal weights equalise the fractions - while letting declared weights trade at the margin.

Scarcity - the mean gap - yields only to producing more, which spends free time. **Cadence** - the variance - is won at **fixed** L by **coordination**: a reserve carries provision from the phase it was made to the phase it is wanted, and **anticipation** fires each rhythm a quarter-period early. So as the federation finds cadence, social desire satisfaction Φ climbs with no one working an hour more - the **coordination dividend**. But timing itself costs - storage pays a spiral toll, and a perishable resists banking at all - so cadence is just one more tension in the field, and **the optimum is the harmony maximum itself**. The buffer a stock carries is the **flat**, un-schedulable fraction of its daily demand spectrum - the part no timing can place:

$$b_r = b_r^0\big(1 - \bar{R}_r^{\max}\big), \quad \bar{R}_r^{\max} = \max_{k\geq 1} \bar{R}_{r,k}, \quad \bar{R}_{r,k} = \frac{|\hat{w}_{r,k}|}{\hat{w}_{r,0}}$$

The full circular-time geometry - Parseval's split of the tension into harmonics, anticipation as the quarter-period operator, storage as phase displacement and decay as a spiral, and the timing optimum that bounds the dividend - is developed in the companion *tau-time*.

Harmony also shapes distribution: the **individually-claimable pool** is allocated to minimise federation-wide harmony loss, prioritising the desires whose shortfall would produce the steepest drop in social desire satisfaction. The citizen's **desired inventory** (live order book) is their priority weights projected onto the individual-claimable pool.

3.2. The credit identity

The **individually-claimable pool** is the set of individually-claimable resources - the portion of the social product set aside for personal consumption. Its labour-content is the total labour-hours embodied in the goods **available to draw** - here the **embodied** measure $\mathrm{SNLT}_{\text{crystallized}}$ is the right one, since a claim should cost what the good cost society: the sum, over every resource in the pool, of the quantity standing above its social portion s_r times its crystallized labour-cost $\mathrm{SNLT}_{\text{crystallized}(r)}$. The social portion is what the scope's own processes will consume this horizon, plus the held-back insurance reserve q_r^{res}: one part of a good serves as fresh means of production and stays social, and only the rest is drawn by members.[17]

$$\underbrace{K}_{\text{pool size}} = \sum_{r\in\mathcal{R}} \underbrace{\max(0, q_r - s_r)}_{\text{quantity above the social portion}} \cdot \mathrm{SNLT}_{\text{crystallized}(r)}$$

When a reserve is released, the social portion s_r falls by that quantity, K grows, and every citizen's drawable credit rescales with it.

K is a sum over the **universal substance** of §Concrete and abstract labour: each $\mathrm{SNLT}_{\text{crystallized}(r)}$ is $\hat{\ell}_r$, the located hours integrated off the lattice by the one value rollup, so the pool is priced in the numéraire every pull and every tension is already denominated in, and a claim on any good in it costs that good's substance.

Consumer voice is a blend of three components, set by social free time:

[17] One portion serves as fresh means of production and stays social; another is consumed by members as means of subsistence. The plan publishes each good's social portion after every solve (`facts/rail.scm`), so K follows the plan and its horizon; before any solve the portion is zero and K is the whole stock.

Contribution - your fraction of total labour. Solidarity - your fraction of total reduced capacity (those who can work less receive more). Citizenship - an equal share for every citizen, scaled by social free time. The three weights always sum to one.

$$v_p^{\text{cons}} = \underbrace{l(1-w)\cdot\frac{L_p}{L}}_{\text{contribution}} + \underbrace{lw\cdot\frac{1-a_p}{\sum_q(1-a_q)}}_{\text{solidarity}} + \underbrace{\frac{f}{N}}_{\text{citizenship}}$$

Consumer voice is simultaneously the citizen's governance weight on consumption decisions and their share of the individually-claimable pool. Claim capacity is consumer voice scaled to the entitlement pool W (defined below): your claim capacity grows as you work, because your fraction of total labour accrues with each matched hour.

$$W \cdot v_p^{\text{cons}}$$

Credit has two readings, kept apart. A citizen's **credit hours** are their validated hours, the gross the work log records. Their drawable credit is their claim capacity less their drawdown, the quantity a claim is checked against. Hours earn the share; the share is what is drawn.

Consumer voice is a **living** quantity. At the start of a cycle, a citizen who has worked no hours carries only the citizenship component - an equal share of social free time. As they work their matched slot, the contribution component accrues, consumer voice grows, and their claim capacity and governance weight grow with it. The **drawdown** d_p is the running accounting of how much of the citizen's claim capacity has been claimed: as the citizen draws goods from the individually-claimable pool, the drawdown rises and the remaining balance, their **drawable credit** $W \cdot v_p^{\text{cons}} - d_p$, falls. Consumer voice stays where work, capacity and citizenship put it.

What a draw leaves. A claim moves goods out of the pool into the claimant's own holdings: K falls by the value drawn, and the whole the shares are shares of stays where it was. The citizens' shares are therefore taken of the **entitlement pool** $W = K + \sum_p d_p$, the pool as it stands together with everything already drawn from it, and

$$\sum_p \left(W \cdot v_p^{\text{cons}} - d_p\right) = K :$$

the credit left is exactly the goods left. A citizen pays for what they take once, through their own drawdown, and never a second time through the pool shrinking under their share.

The **capacity to labour** (between zero and one) is a **governed assessment**: set by a governance-sealed act and issued as a signed credential naming its assessor, its moment and its reason, so a judgement about someone's capacity is attributable and contestable rather than anonymous. The design intent is that it be **attested** by fellow workers through capacity-recognition — a production-governance act excluding self-attestation, weighted by producer voice, giving the labour-weighted average of others' attestations — and that aggregation is not yet built. The **aggregate incapacity** is the average gap between full capacity and each citizen's actual capacity. The **labour share** is one minus social free time.

$$\underbrace{a_p}_{\text{capacity to labour}} = \frac{\sum_{q\neq p} L_q \cdot x_{q,p}^{\text{cap}}}{\sum_{q\neq p} L_q}, \qquad \underbrace{w}_{\text{aggregate incapacity}} = \frac{1}{N}\sum_q (1-a_q), \qquad \underbrace{l}_{\text{labour share}} = 1-f$$

Convergence. As social free time approaches its maximum, the contribution and solidarity weights vanish and citizenship becomes the whole of consumer voice - equal for every citizen. Claim capacity becomes an equal share, W/N, of the entitlement pool for everyone, and drawable credit that share less each citizen's drawdown - **from each according to ability, to each according to need**.

3.3. Credits and external liquidity

Consumer voice is a personal, non-negotiable position - it belongs exclusively to the citizen who earned it. The drawable credit $W \cdot v_p^{\text{cons}} - d_p$ has use value within the commons alone.

The federation coexists with commodity economies. It may need to acquire external assets - dollars, euros, cryptocurrency, raw materials available only on world markets, patent licences, server capacity denominated in fiat. Any such asset enters the commons as a resource specification with a crystallized SNLT that reflects the **social cost of acquisition**: the total labour the federation must expend, by whatever means, to obtain one unit. If acquiring $1 000 requires 20 hours of labour (through export, through service contracts, through whatever interface the federation maintains with the commodity economy), then the crystallized SNLT of $1 000 is 20 labour-hours - and that cost appears as area on the surface like any other resource.

Citizens can therefore claim external assets with their credits, at the price denominated in crystallized SNLT. A citizen spending 20 hours of drawable credit on $1 000 is spending exactly what the community spent to get it. The flow is **one-directional**: credits purchase external liquidity. Voice, credit, and standing in the commons are earned only through participation. The boundary is absolute - the commons is a **sink** for external assets, converting them into social use at their labour-cost of acquisition.

This generalises to any external asset class: fiat currencies, cryptocurrencies, commodities, digital services. Each enters the plan as a resource the federation must produce or acquire; its crystallized SNLT is set by the value rollup over the recipe for acquisition; and citizens draw against it with the same consumer voice they use for bread or shelter. The commons metabolises the commodity economy; the boundary admits resources inward and keeps the internal accounting sovereign.

3.3.1. Three doors inward, one door out

Money reaches the commons through three doors, and each preserves the same boundary. **Acquisition**: the federation expends its own labour to obtain the asset, and the crystallized SNLT of that acquisition is the asset's one internal price - the door already described above. **Gift**: a citizen socializes a purse - a consume grant of their own money into a scope's use-pool. The grant is a standing promise, held as a credential: up to the granted quantity is the plan's to spend, nothing moves until the plan draws it, recall returns the unspent remainder, and nothing is ever owed back - a gift carries no return leg. **Restitution**: a citizen fronts their own money for an acquisition the commons wants, and what they advanced returns to them - exactly, once, and in money. The good is raised into the commons the moment it is recorded; the citizen signs a claim over what they gave; the commune's own signature triggers a debt from that claim; and the payout settles the debt from the commons' pooled liquidity at the crystallized-SNLT ratio. The whole chain is ValueFlows end to end: the raise fulfills the plan's own commitment for the good, the claim is `triggeredBy` the raise, the debt `triggeredBy` the claim - provenance on every link, and no agreement object,

because a past fact entitles a reciprocal where only mutual promises need agreeing.[18]

The out-door stays single: credit converts to money, and money converts to nothing internal. A citizen restored by restitution ends exactly where they began - the debt names the advanced amount and no more, with zero interest as structure rather than policy: the credential carries no rate field to set. And between gift and work stands nothing: the commons carries **no lending primitive**. A loan of commons value would fund production the commons cannot see - wage labour hired outside, paid less than it produces - and return as clean, recipe-priced goods; the seam that credit's own construction closes (credited hours are hours actually worked, with no channel for anyone else's uncompensated labour to enter) would reopen through property. Whoever wants value from the commons works for it; whoever wants to give, gives; whoever fronts is made whole.

Each door is also a crossing of the legal exterior, and the membrane is where those crossings live (§The catalogue of crossings). Acquisition by sale or service contract is a sale to the public. Every passage between credit, key-held value and state money - a purse granted in fiat, a restitution paid out, a citizen drawing dollars on credit - is a conversion, tripped once per event at the one adapter that faces the bank, and it authenticates the person who crosses. Inside the membrane a member draws on credit and nothing is sold or converted; at it, the federation sells and converts in the open, and the bill for each passage is the ordinary fold of the legal seam.

4. The federation and its members

The structure that carries the plan - a tree of scopes under the Universal Commune - how labour is matched into its work, and how membership is held.

4.1. The federation and the Universal Commune

The federation is a tree of scopes. Each scope is a **branch of productive activity** - food production, health care, education, construction - with its own governance, its own plans, and custody over the **social property** entrusted to it. Social property is the means of production held in common: tools, land, infrastructure, materials. Custody means a scope tends and stewards the social property it uses; ownership remains with the association as a whole. Non-leaf scopes hold custody over child scopes, aggregating governance upward. At the root sits the **Universal Commune** (UC): formed at planning-time, its children are all **federation roots** - scopes with no parent. The UC holds the offered availability of the entire association - the union of every citizen's availability - as one pool. **Material is local; labour is universal.**

A producer holds skills, offers time, and the matching places them into whichever branch of activity fits. Scopes are branches, and producers move freely among them. Voice is **radically local**: a producer's voice in scope s is grounded only in labour they have actually contributed to s's social-intents. Working in food production confers voice there and upward through parents to the UC; it confers no voice in manufacturing. Anyone may work in any branch, and no one accumulates political rights detached from where they actually produce.

[18] Restitution's domain is **unsocialized** money alone. Money already socialized owes nothing back - the gift settled at grant time, by whoever gave it. A claim that names a consume-granted purse as its source is refused outright; one that names the citizen's own ungranted balance clears the check; one that names nothing surfaces the citizen's standing purses beside the claim, and governance weighs the two - the mechanism informs, it never deducts on its own, because an unrelated purse and a genuinely-fronted purchase are indistinguishable from outside.

A personhood credential answers a different question from voice: it says **who counts** - the federation's own N, the citizenship component every recognised person carries into consumer voice universally, from the moment of recognition. It does not say **where they may act**. A production-governance act - accepting a grant into a scope's use-pool, accepting a reimbursement claim against it, any decision that binds a scope's own social property - answers to that scope's producer voice alone, the labour-voice described above: grounded only in labour actually contributed there. Personhood recognised by one ecosystem carries no standing to act for a scope a person has not worked in - the same locality that keeps voice itself radical.

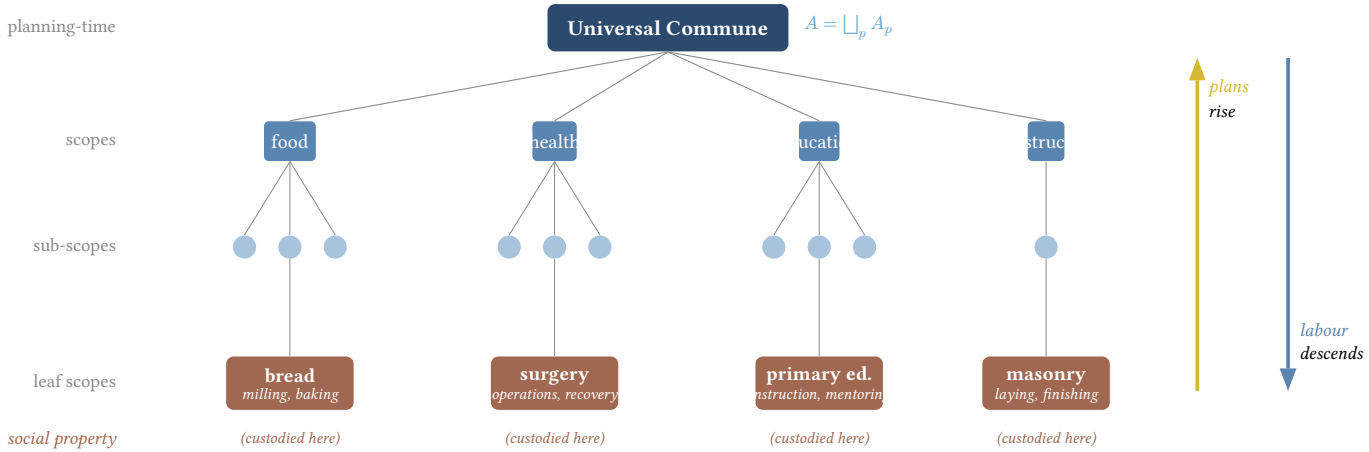

Figure 6: **The federation scope tree.** The Universal Commune sits at the root, holding the whole pool of offered availability A. Below it, scopes branch into sub-scopes and leaf scopes - each a branch of productive activity. Social property (means of production) is custodied at each leaf. Two flows meet at every level: **plans rise** from leaves upward, carrying work-intents; **labour descends** from the UC as matched hours, placed wherever the work-intents fit.

4.1.1. Plans rise, labour descends

Two flows meet at every leaf. **Plans rise**: each leaf, custodian of its tools and material stocks, puts forward a plan - a bundle of work-intents, each specifying a skill, a time window, and the hours required. Plans propagate upward through the scope tree to the UC. **Labour descends**: the UC draws from the whole pool of offered availability and deploys it into the leaves whose work-intents fit.

Plan acceptance is a production-governance act - weighted by labour-voice alone - and it is made piece by piece (*The voice field*): a scope's producers shape the plan through their stances on its parts, hold what they refuse outright as a wall by ballot, and **verify** the certified plan that results as a valid use of the scope's social property and labour allocation. Reach follows the solve that placed the hours: hours a parent scope's solve places are recognised at the parent, and its children are inside that plan's **acceptance domain**; hours the UC's matching places are recognised across the entire federation.

4.1.2. Credits and acceptance domains

A citizen's claim capacity ($W \cdot v_p^{\text{cons}}$) accrues through validated labour. Its **reach** is the reach of the solve that placed the hours. A plan P is solved at some scope over the scopes beneath it, and that span is its **acceptance domain** $\text{accepted}(P)$: hours matched within P earn credit redeemable

against the individually-claimable stocks of every scope in accepted(P). Credit accrued in food production under plan P becomes spendable on goods from manufacturing if and only if both scopes lie within accepted(P).

Reach rises monotonically with the solve: a leaf that plans for itself alone reaches the leaf; a parent's solve, placing hours against the wants and roster of all its children, reaches the parent and everything inside it; the UC's union matching places hours against the whole federation's wants and roster, and the credit is **universally redeemable**. This is how credits become federation-wide - by being placed by the federation's own solve, an hour's reach read off the provenance of the commitment it fulfils (*The voice field*, "What acceptance becomes"). Credits remain non-transferable, non-negotiable, and personal; they lapse on loss of citizenship.

4.1.3. Refederation

Scopes can **recompose, split, and merge**. The trigger is metabolic coherence (the share of a scope's flows that stay inside it - *Metabolic coherence*, below) measured on live flows: when the planner detects that a split or merge would raise metabolic coherence, it auto-generates a proposal. The affected scopes accept or refuse it by ballot, a structural act weighted by labour-voice - **the flows are computed; the redrawing is decided**. Resources flow between scopes through the plan itself: one scope's output becomes another scope's input, linked by VF resource flows under standing agreements. The metabolic coherence measure tracks whether these cross-boundary flows are structurally necessary or a sign that scope boundaries should be redrawn.

What makes this a good instrument is that it is **local**. Each scope can compute its own $F_{\text{internal}}/F_{\text{total}}$ from flows it already sees, and the quantity it thereby improves is a **global** one it cannot see: how far a pull must travel before the federation agrees. Coordination settles by moving boundary information **one hop per round**, so what costs rounds is the **diameter** of the cover — the longest chain of scopes a good must cross to reach its eater — and no scope is in a position to measure that. Coherence stands in for it by attacking its dominant cause: a long cut forces a good across every intermediate boundary, so each middle scope becomes mostly a conduit, and being a conduit is exactly what a low $F_{\text{internal}}/F_{\text{total}}$ registers. On one economy cut two ways, the chain moves 9,806 units across boundaries where the star moves 1,283 (§Inter-scope propagation) — so the local measure does prefer the cheap cover, and it does so without any authority computing graph diameters on the federation's behalf.

It is a **proxy**, and worth knowing where it is blind. Coherence counts **volume**; diameter counts **distance**. A ring and a star can move the same quantity across their boundaries at diameters $S/2$ and 2, and coherence cannot tell them apart. The shape that slips past is therefore a federation whose flows are modest but **strung out in a line** — each scope tidy and self-contained, the cover as a whole long. That is a governance question rather than a computational one: a federation that wants both should read the **shape** of its cover, not only the tidiness of its parts.

4.2. Matching

The matching assigns available labour to accepted plans in two layers.

4.2.1. Federation matching: plans to labour

Each plan carries a scale (between zero and one - how much of the plan runs) and a bundle of work-intents, each specifying a skill, a time window, and the hours required. The binding constraint is

cumulative labour: the total labour demanded by all scaled plans must not exceed the federation's total offered availability.

$$\sum_P r_P \cdot x_P \leq A$$

The algorithmic planner maximises voice-weighted plan satisfaction subject to this constraint - a **convex quadratic program** for continuously-scalable plans, or a **mixed-integer program** when skills carry irreducible atomicity (a surgery cannot be 0.3 surgeries; a teacher cannot occupy three classrooms at once).

Plans are **atomic**: their work-intents stand or fall together. When labour falls short, the planner scales plans down, selects among variant recipes, or returns residual capacity for re-planning. The planner discloses **alternative budgets** - the labour that would free up under different matchings - so leaves can re-author and counter-propose. A leaf whose counter-proposal beats the marginal incumbent enters the next round; the iteration terminates because re-drawn plans are strictly smaller in labour-demand. Convergence occurs when no leaf can improve within **any** budget the federation can offer.

4.2.2. Egalitarian matching: producers to slots

Within each accepted plan, a second layer assigns individual producers to specific work slots. Each work-intent specifies a skill; the matching draws from the subset of citizens who hold that skill and sets preference weights **uniformly** among them: every qualified citizen gets equal weight. This is the **egalitarian matching principle** - equal standing among all who can do the work. A citizen's own preference over their slots - eager here, reluctant there - is a statement on this layer, read the same for every citizen whatever their hours: clamped to one governed range and centred on the citizen's own mean over the skills they hold, so it reorders their own assignment among the willing and moves nobody's total load. Hours buy voice on **how** a part is done (*The voice field*) and never assignment.

How the work is done is shaped by contribution; **what** the community needs is shaped by all three voice components; **who makes which part** is determined by citizenship. Rotation, cross-skilling, and access to every kind of work are available by right of citizenship.

The matching produces a single accounting primitive: matched hours worked per citizen per scope. From this, everything derives: a citizen's share of scope labour is their producer voice; their share of federation labour is their federation voice; and the share of total social time beyond planned labour is social free time.

$$\underbrace{L_{p,s}}_{\text{matched hours in scope}} \longrightarrow \underbrace{v_{p,s} = L_{p,s}/L_s}_{\text{producer voice}}, \qquad \underbrace{L_p}_{\text{total matched hours}} \longrightarrow \underbrace{v_p}_{\text{federation voice}}, \qquad \underbrace{f = 1 - \Lambda/(NT)}_{\text{social free time}}$$

Both layers fold into the one solve. These two matchings are described here as stages because that is how they are **governed** and **recognised** - but in the unified engine they are not separate computations. As §"Labour is a good on the lattice" shows, the **federation** layer's constraint ($\sum_P r_P x_P \leq A$) is just the located labour **balance** (one per skill and cell, not a single global pool), and the **egalitarian** layer's uniform weights are the convex **EMP congestion-tension**. Equal standing reads as equal **fractional** effort - the congestion equalises each producer's share of the time they offer, so all who can do the work give the same fraction of their availability, the same move toward equal free time - and it is sized to settle **who** works while the located balance and the free-time pull decide **how much**. Solve the located program and the producer-to-slot assignment - the matched hours $L_{p,s}$ above - is

read off the worker-column levels; the planner emits it in VF's two stages - a work **intent** per process-need (what is intended) and a work **commitment** per worker that satisfies it (who is assigned). The separate bipartite matcher remains for the **decentralised** mode (peer-to-peer free association, atomic/indivisible work) - where there is no global solve and a heuristic match is the right tool - but the cooperative plan needs neither stage as a stage.

4.2.3. Matching as validation

Plan acceptance validates the work as socially necessary; the matching validates the assignment. Three moments suffice:

Plan acceptance. The scope's producers stand behind a plan - a bundle of work-intents, each specifying a skill, a time window, and hours required. This is the political act, made piece by piece: weighted by labour-voice at the grain of the part, the field of stances says how each piece is done, the wants say the work is worth doing, the recipe's map and its workers' contrary observations say the hours are well-sized, and the scope verifies the certified plan that results.

Matching. The planner assigns citizens to the accepted work-intents. A match occurs when $k \in S_p$ and the citizen's offered availability intersects the demand. The match **is** the allocation of work, voice, and credit simultaneously.

Time. The citizen works their matched slot. Hours enter L_p as they elapse, accruing gradually as the slot is lived through. The work and the ledger are directly connected.

In a community of associated producers, labour manifests itself as simple, useful labour - useful in the sense that its end is the production of use-values. Useful labour is labour which produces the intended useful result. The plan determines the **intended** result: the resource specification, the recipe, the skill requirements - all set before anyone is matched. The plan **defines** what useful labour is for this cycle.

Output quality is a function of the recipe (was it well-designed?), the skill match (did the plan assign the right skills?), the training (did skill (re)production development invest enough?), and the means of production (are the tools adequate?). These are all **plan-level** responsibilities. The individual contributed their time where the community directed it; when the useful result falls short, the plan needs revision - better recipe, better training, better matching. Quality is a **planning** question. The correction lives in the next cycle's plan, and the feedback loop is collective and structural.

The plan sets the specification; administration reviews the outcome. Between these two moments, the work is free.The administrative overhead diminishes as the federation develops.

Visible non-contribution - absence, idleness, obstruction - is handled by the scope's ordinary self-governance: a labour-voice-weighted decision to revoke accrued hours, the same mechanism as any other production-governance act. The criterion is whether the person did the work. Willful sabotage falls here; genuine incompetence is a planning problem (better training, better matching). **The normal path: you are matched, you work, the proportions flow.**[19]

[19]The functions traditionally assigned to a legal system are already distributed across the plan. Conflict resolution, restorative justice, mediation, care practices, healing, safety measures, prevention, rescue services, emergency response, child protection, elder care, disability support - each is a branch of productive activity with its own scopes, skills, recipes, and matched hours. Administration verifies intent against specification. Scope-level self-governance handles visible non-contribution. The protection of persons from harm - violence, abuse, coercion - is treated as a specification: bodily safety is a condition of citizenship, and care and intervention scopes have standing to act when that specification is violated, including across scope boundaries. The federation's desire for safety enters through priority weights over communal provision; the intervention itself is matched labour (mediators, care workers,

4.3. Citizenship

Citizenship in the federation is standing in the association itself. A citizen carries the citizenship share the citizenship component of consumer voice (f/N) from the moment they enter. The N that divides is a count of **persons**: personhood is witnessed into the federation as a credential, and every equal share - the citizenship component, the free-time dividend - divides among witnessed persons. A key is an instrument; a citizen holding several keys is one citizen.[20] Children are citizens from birth: their citizenship component is active immediately, and their reduced capacity ($1 - a_p$) enters the solidarity component, attested by producer voice as for any citizen unable to contribute.

Voice and credits both lapse on **loss of citizenship** - the structural form in which death and any other terminal exit from the association is expressed. While one is a citizen, accrued voice and unspent credits are simply theirs; on ceasing to be a citizen, both fall away with the standing they belong to. The asymmetry between them is the point: **voice is a standing** - it persists as long as citizenship persists, heard without being diminished; **credits are a stock** - they are consumed once the claim is met. One labour event grants both an incremental say in what the federation does and a one-shot claim on what it produces. The two move on different rails from the moment they are minted.

5. The living plan

How the plan, once set, actually runs - stocks drawn down and replenished in real time, each tension weighed against the limit it nears.

5.1. The surface as a stock network

Most of what the federation holds on hand is a **stock**: a quantity it wants to keep at a target level.[21] Skills are stocks of trained people; Means of Production are stocks of tools and infrastructure; durable and buffered consumption goods are stocks of usable product. Linked by the recipes that draw on one stock to reproduce another, these form **one federation-wide stock network**: every node carries a desired quantity, drawdown signals propagate through recipes, and replenishment is pulled by actual consumption rather than pushed by forecast. Services - a verification, a consultation, a lesson - hold no stock; they are reproduction labour drawn directly through the network, appearing in what follows as demand that draws stocks down, bounded by the skill and facility stocks they pass through.

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emergency responders). Environmental regulation is already structural: ecological specifications (regenerative envelopes, absorptive capacities) enter every plan as hard possibility-tensions, and administration verifies the draw the same way it verifies any other specification. The plan organises all productive activity; there is no remainder that requires a separate legal apparatus.

[20] `person-aid?` (`observation/account.scm`), read off the live First-Person personhood credentials (`openvtc/first-person.scm`) wherever citizens are counted - the credit fold's citizen roll, the dividend, social free time's N - and failing open until the first person is recognised. This is what makes consumer voice additive across keys, the condition the split-invariance theorem rests on (`SolverCertificate/Incentives.lean` `split_no_gain`).

[21] This pattern - strategically-sized stocks at decoupling points, replenishment pulled by actual consumption rather than pushed by forecast, stock size as a statistical function of drawdown rate, lead time, and observed variability - was articulated in operations literature as DDMRP (Demand-Driven Material Requirements Planning). See Carol Ptak and Chad Smith, *Demand Driven Material Requirements Planning* (2011). DDMRP renders stocks with three discrete zones (green, yellow, red) because firms in a market face per-order transaction costs, minimum order quantities, and volume-discount incentives that reward batching. The federation has none of these frictions and uses a continuous formulation throughout - the same statistical logic without the batching artefacts.

Each stock carries four sizing inputs, each with a different temporal character:

- **Consumer voice** (top-down, slow): priority weights aggregated into social desire set the relative size of stocks across the catalogue.
- **Drawdown rate and variability** (bottom-up, continuous): how fast the stock is being consumed on average, and how spiky that consumption is - both computed directly from the ledger of economic events, purely statistical.
- **Replenishment lead time** (engineering, semi-static): how long it takes to refill, from recipe duration plus upstream chains. Long lead times mandate larger stocks.
- **Ecological and capacity caps** (top-down, hard): regenerative envelopes and total availability A cap any stock that would breach them.

Plans are composed **in real time**. As consumption draws a stock down, its **gap** - the proportion below target - rises continuously from zero. The scope generates a replenishment intent sized to the gap: a stock at 60% of its target produces a request to refill the remaining 40%.[22] The intent decomposes through the recipe into work-intents and material-input-intents. Material-input-intents become drawdown signals on upstream stocks, propagating through the recipe graph until they hit raw materials or imports. Work-intents enter the matching queue, get matched to producers under the egalitarian principle, become commitments, are worked, and fire economic events. Each event updates stock state. Each state change can trigger new intents. The loop runs on.

A stock is drawn down in one of two ways, and the way determines what closes the gap. A means of consumption is **consumed** - destroyed in use, its quantity falling directly, so the gap is closed by **producing more** (baking replaces eaten bread). A means of production is **used**: the oven persists and **wears**, shedding a fixed fraction δ of its condition with each use - a physical quantity, set by the recipe and scaling with throughput. That one wearing feeds three distinct measures, and they part ways:

on the oven	scales with	denominated by
maintenance labour	use × wear δ	the **repair** recipe's $\mathrm{SNLT}_{\mathrm{direct}}$
depreciation in value	use × wear δ	the oven's $\mathrm{SNLT}_{\mathrm{crystallized}}$
capacity pull	downstream demand and scarcity	a dual - met by SNLT only to decide expansion
maintenance urgency	the bread it holds up	a derived pull

Wear is not an accident of use; it is the shadow of use, its constant twin. **Maintenance** rebuilds the worn condition - its own repair recipe, sized by the wear (so by use) and costed at **that recipe's** direct labour, a patch cheaper than the build. **Depreciation** is the oven's own embodied labour shed into the bread - δ of its $\mathrm{SNLT}_{\mathrm{crystallized}}$ per use, carried by the value rollup into the product's cost - reckoned at the oven's **cheapest current reproduction**, built afresh or repaired, whichever is socially necessary, so where repair is the cheaper way to hold condition it **lowers**

[22] The continuous gap signal $g = (\text{target} - \text{current})/\text{target}$ replaces the discrete zones of classical operations practice. In a planned economy the market frictions that justified batched ordering do not exist: replenishment is a signal within the plan rather than a purchase order, information is transparent across scope boundaries, and the only physical constraint on quantity is recipe granularity (the kiln that fires in batches), which the recipe handles separately. The continuous gap gives the planner richer information (one stock at 37% gap, another at 12%, is finer than "both yellow"), allows proportional replenishment under upstream scarcity, and mirrors the continuous harmony function used at the federation level. See `docs/planning/solve/harmony-planning-design.md` for the full derivation.

the depreciation each loaf carries: longer life and lower per-unit charge are one fact. And the oven's **pull** is the marginal harmony one more oven-hour would buy when its capacity binds - a possibility-tension dual set by the bread it gates, met by $\mathrm{SNLT}_{\text{crystallized}}(\text{oven})$ at the single question of whether to build another. Neglect prices itself: an unmaintained oven's rising barrier is a bill the plan reads **before** the breakdown, not the wreck it reads after. So the `use` action, repeated, is a slow continuous drawdown, and maintenance its continuous replenishment - pulled, like any derived demand, as loud as the bread waiting behind it.[23] Skills draw down the same way - a `work` flow exercises a trained person, who persists; the standing maintenance there is recertification and refresher practice, the labour that keeps a skill from atrophying.

Inspection earns its own line in that upkeep. A conformance edge - a producer still holding electrician, an oven still in spec, a batch still meeting the food-code - is a claim that **drifts**: skills atrophy, machines fall out of calibration, at a rate κ. A possibility-tension is only defended if its slack is **known**, so knowing it is labour - the recertification that finds the skill sound, the survey that reads the wear - scheduled on the one lattice like any other `work` flow, routed to a qualified verifier, spread by the same egalitarian matching, drawn from the one SNLT budget it competes with production for. **How often** to re-verify each edge is set at the margin, by the plan's own reliance on it - the load D it routes through the edge, the harm a silent miss would do, its dual in the solve. Where a check is a fine intensity the cadence is $a^* = (1/\kappa)\ln(\kappa D/\ell)$, clamped, rising with reliance D and falling with the check's labour ℓ; where a check is a lumpy **event** of fixed cost K - convening the auditor, the exam apparatus - it batches into a periodic $T^* = \sqrt{2K/(D\kappa)}$, run less often but fully as K rises. Either way inspection **recedes** from an edge the plan barely leans on, or has well-substituted so its reliance splits across the skill-closure, and presses onto the load-bearing one. And the interval the plan derives is the credential's recertification date - a heavily-relied competence takes a short recert, a marginal one an open credential - carried in the skill-conformance credential's `valid-until`, the one field administration reads to decide whether the edge is still live. The plan derives the interval and a tested function stamps it onto a credential (`apply-recert-schedule`); the live path does not yet call that function, so the interval is reported in `conformance-recert` and stamped only where a caller applies it. The barrier's steepness is not an engineering constant but a governed profile in the fragility catalogue, ratified like any planetary boundary.[24]

A stock's on-hand figure is an edge of the same kind, and the check that keeps it known is a **count**. Its interval is the same T^*: the reliance D is the good's crystallized labour over the larger of what the plan draws in a period and what the ledger holds, and the drift κ is read off what earlier counts found the ledger off by. A count is laid as a commitment on the calendar beside the solve, offered

[23] When wear outruns repair - the machine fails outright - the scope falls back on the **insurance reserve** (the stand-by capacity in q_r^{res}) to keep producing, and rebuilding that reserve becomes its own gap. So a single breakdown can open three gaps at once: the lost output, the maintenance of the broken unit, and the drawn-down reserve.

[24] The full arc is `docs/legal/inspection-in-plan.md`, realized in `module/playnet/planner/formulas/cadence.scm` (`edge-reliance`, `inspection-intensity`, `recert-interval-discrete`, `reliance->valid-until`, `apply-recert-schedule`), with the confidence-barrier steepness a governed row in `conformance-catalogue.scm`; `plan-scope` emits the recert schedule off a live solve as its `conformance-recert` field - for skills and for the means a process uses - and with `#:inspect` schedules the verifier's hours in the solve itself: every edge a scope's recipes lean on becomes a `Verif|` freshness stock, worn per run, restored by an inspect recipe, its margin a band floor (`solve/inspection.scm`, `tests/planner/test-inspection-live.scm`). The proofs run from the single-edge a^* oracle (`test-inspection-cadence.scm`) through the in-plan multi-period solve (`test-inspection-in-plan.scm`), the worker-scheduled path (`test-inspection-scheduled.scm`), the dual closure that reads D off the plan (`test-inspection-dual-closure.scm`), the discrete interval (`test-recert-discrete.scm`), and onto the credential (`test-inspection-credential.scm`, `test-recert-apply.scm`, `test-conformance-recert-live.scm`).

to a counter in turn; its hours enter the solve's own labour budget once a count recipe states them, which remains to be built.[25]

There is no plan document. The plan is the live stock configuration plus the matching queue plus the commitment ledger. Governance tunes the stock network: consumer voice adjusts target levels and catalogue composition; producer voice tunes recipes and scope structure; UC-level endorsement adjusts cross-scope constraints. The plan emerges from the tuning.

5.2. Inter-scope propagation

Every flow between scopes is mediated by stocks at the boundary. A bakery holds an **input stock** of flour; the mill that supplies it holds an **output stock** of flour. The two are linked by a VF **transfer** relation - mill is the provider, bakery is the receiver - and each scope governs its own side of the boundary independently. Because every scope is itself a decoupling point - each is a branch of productive activity with its own custody, governance, and recipes - the boundary stocks fall out of the scope structure naturally rather than needing to be installed separately.

A concrete cascade:

1. A bread recipe fires at the bakery. Its consume-flour flows draw on the bakery's flour input stock. The level drops, recorded as a consume event on the ledger.
2. The stock's gap rises above zero. The bakery generates an inter-scope replenishment intent proportional to the gap: **transfer 1 000 kg of flour from the mill scope**. The intent reaches the mill's order book.
3. The mill fulfills from its flour output stock. A transfer event fires: provider mill, receiver bakery, quantity 1 000 kg. The mill's output stock drops; the bakery's input stock rises.
4. The mill's own output gap rises and triggers its production cycle. Wheat input drops; if its gap rises in turn, an inter-scope intent goes to the farm scope. Cascade continues upstream until it reaches raw materials, ecological draws, or imports.

Each scope manages its own stocks from its own local signals, and the **real-time** inter-scope coordination happens entirely through the order/fulfillment cycle: drawdowns cascade **upstream** as intents; fulfillments cascade **downstream** as transfers - event-driven, continuous, with no scheduler in the loop. The plan that sets the targets this cascade aims at *is* solved - globally and cheaply (§One program), and decomposed by scope only when convenient (a single solve over the whole federation is tractable). The cascade is the fast layer beneath the plan, not a replacement for it. Reserves are the slack it spends: a shortfall or breakdown draws on the insurance margin and pulls replenishment directly, with no plan revision - and a held-back quantity can be **re-routed across rails** (an insurance reserve, or a stock kept as means of production, released into the individually-claimable or communal pool), rescaling the pool and every citizen's drawable credit (§The credit identity) without queues or stranded claims - two redundant degrees of freedom beneath the coarse plan.

Several mechanisms keep the cascade stable:

Decoupling at every scope boundary. Each inter-scope stock absorbs variance instead of transmitting it. Downstream consumption fluctuations raise or lower a single stock level, which

[25] `count-interval`, `stock-reliance` and `drift-rate` in `module/playnet/planner/formulas/counting.scm`, over `recert-interval-discrete`; the door and the schedule are `module/playnet/planner/doors/count-door.scm`; the design and its open rows are `docs/observation/capture.md` §4 and `docs/observation/README.md`.

the upstream scope sees as one slowly-changing signal rather than a stream of spikes. Scopes **are** the decoupling points; no separate decoupling layer is needed.

Variability-driven re-sizing. Each stock is sized as a function of observed drawdown rate, lead time, and variability at its boundary. When variability rises, the stock target grows; when it falls, the target shrinks. The system tunes itself to the actual statistical character of the relationship.

Recipe alternatives and substitute inputs. When a supplier scope cannot fulfill (output gap large, long lead time), the consuming scope can switch to an alternative recipe using different inputs, or source the same input from another supplier scope. The recipe library is a network of alternative pathways, not a fixed graph.

Metabolic coherence $M(s)$ as engineering parameter. High-M scopes have tight internal loops and transmit little variance across boundaries. Low-M scopes are highly coupled and carry more of the cross-scope coordination load. Scopes can be split, merged, or restructured to shift M toward operational targets.

Multi-supplier resilience. A consumer scope may carry orders against multiple supplier scopes; a supplier scope may serve multiple consumers. Order allocation under contention uses harmony weights - the planner favours fulfillments that maximise social desire satisfaction subject to capacity.

When local mechanisms cannot absorb a disruption - a stock whose gap persistently stays large, a supplier that cannot meet lead time even with alternatives - the signal escalates to the UC level. UC-level coordination can re-allocate labour from the federation pool, endorse plan amendments, or invoke ecological/import strategies. The escalation path is structural, not exceptional: federation-wide harmony optimisation is one of the UC's standing functions, invoked whenever local scopes cannot resolve coherence at their own level.

The cascade is denominated identically across the surface. Each transfer event carries crystallized SNLT (labour content) and substance quantity (kilograms of flour, kilowatt-hours of energy). Both dimensions update both stocks simultaneously - labour metabolism and material metabolism on the same primitive. The federation's coherence is the coherence of **both ledgers** propagating together through scopes.

5.3. Shared budgets and possibility-tension

A single ecological agent - a watershed, a forest, a fishery, the atmosphere - is typically drawn on by **many** scopes: agriculture, manufacturing, household consumption, energy. Its regenerative envelope is **one pooled budget** that all consumers share. The budget itself is a stock on the surface, with its own gap signal, held in **stewardship custody** at the scope that speaks for the agent. Every consume or use event from any scope is debited against this one shared stock.

When pooled draws across consumer scopes approach the envelope, the budget's gap rises - and with it the agent's **possibility-tension**: the strain, in labour-hours, of pressing this limit of the possible. It is a **hard** tension: it diverges as the draw nears the envelope, so its **pull** climbs steeply and, near the limit, out-pulls every desire-tension no matter how loud. New commitments are priced out before the envelope is breached. The pooled budget catches the distributed-draw failure mode in which many scopes each individually stay within their share, but collectively exceed it - the architecture that defeated every previous attempt at coordinated resource management.

This is the general shape of the federation's **harmony** (§1): one field. Harmony maximises by relaxing desire-tensions while pulling away from **possibility-tensions**, both denominated in labour-hours on the one surface.[26] A soft desire-tension can always be traded against another; a hard possibility-tension, near its limit, cannot be traded away by any priority weight - which is why no desire can buy a draw above an ecological cap, an over-commitment of labour beyond A, or a surgery without a certified surgeon.

Two of those families differ in a way worth naming. A hard limit carries the **swept** weight, so it is barely felt until its slack is nearly gone: the plan rides to the edge, which is exactly right for an accounting identity — all the flour **should** be used. A fragility profile carries a **fixed** weight, so it is felt at the optimum and the plan holds off while holding off is still cheap. The margin is **inside** the barrier, and no desire can spend it.

The biosphere and human reproduction both hold their precaution that way. A subsistence floor states what a plan may never do, so it carries its margin inside the barrier exactly as an ecological envelope does — §Reproduction labour puts calamity insurance "in harmony as a **hard** possibility-tension rather than a soft desire", and the floor is where that lands. A margin carried in the reserve tranche would instead be a low-priority desire, and so the **first** thing shed when resources bind: structural precaution is precisely the property that keeps a floor out of that class.

The shape a floor wants **diverges at bare need** — so nothing crosses it — and **satiates at the margin** — so nothing is over-made for it. One curve does both: $w(1-u)^2/u$ for $u = s/D$, which meets an ordinary desire in value, gradient **and** curvature where the margin is reached, and parts from it as satisfaction collapses. Scarcity therefore erodes the margin and leaves the floor: on one economy it holds 119 with labour to spare, 107 once labour is short, and never breaches 100, where the same plan carrying no floor stops at 100 exactly.

A log barrier belongs on a **ceiling**, where its pressure and the free-time term push the same way — draw less — so the two settle against each other. On a **floor** it pushes supply upward with only thrift resisting and equilibrates at w/κ: a number about the weight rather than about the margin, which places a floor asked to hold 120 at 283.

The same curve answers a second question. An ordinary desire-tension is **bounded**: a wholly unmet want costs w_d and no more, so a want priced far above the rest can be set aside for a finite price, while promoting it to a hard floor makes it all-or-nothing. This shape is the middle term: it cannot be bought out at any finite price, and in the ordinary range it remains an ordinary want. It protects a **class**: an unbounded penalty always outbids a bounded one, so one of these placed beside ordinary desires consumes the budget and the ordinary desires give way — the protection is meaningful applied across a whole class of needs, and used that way it holds the worst-off up.

[26] Operationally $\mathcal{H} = -\sum_d D_d - \sum_a P_a$, where the desire-tensions $D_d = w_d^{\text{desire}} \cdot g_d^2$ are bounded (quadratic deficit) and the possibility-tensions P_a are barriers that diverge at each limit - total labour $L \to A$, ecological envelopes, capacity, skill redundancy, regulatory gates. Formally this is a barrier method: the divergent P_a play the role the hard constraints played in the convex programme of `docs/planning/solve/harmony-planning-design.md`, and the **pull** of a tension is the gradient of its term - the dual variable, for readers who want the linear-programming correspondence. The hard cap is simply where $P_a \to \infty$. Two barrier families share this divergent shape and differ only in their weight: the truly-**hard** constraints (material balance, $L \to A$, a **forbid**) carry the **swept** weight $t \to t_{\min}$ (swept toward zero, floored there in practice — §7.11), so they bind only **at** the wall (ridden to the edge); the **fragility profiles** (the ecological envelopes and the catalogue below) carry a **fixed** governance weight w_a, so they are felt **at the optimum** — which is precisely what lets the federation ease off **before** the limit by its own arithmetic rather than merely halt at it.

One last distinction, which is about the engine rather than the economy. A solver erects tensions of its **own** — a proximal term holding scopes together while pulls converge, a cap pinning a crossing while a plan is reconciled. They wear the same shapes and make a different kind of claim, so tensions of the **method** are marked as such where they are built, and the rule is structural rather than remembered: **the solver sees everything, the certificate sees only the world.** A method tension may steer the search; it may never appear in what a plan claims to have proven. The distinction earns its keep twice over: certifying against the proximal-augmented program would credit a scope for satisfying scaffolding the coordinator erected, and reading a reconciliation cap as a real limit sends the bound to infinity — a cap sits at slack ~ 0 by construction while a scope's own barriers sit far from theirs, and one central-path t cannot fit both. Marking at construction means a change of coordinator carries its own scaffolding with it.

Where harmony does its daily trading is among the **soft** desire-tensions: when two desires both sit well clear of any limit but collective labour cannot satisfy both, the quadratic deficit shares the shortfall by priority - bread carrying the heavier social desire and pastries the lighter split the baker-hours, neither at zero, the heavier weight absorbing less of the deficit. The **hard** work is at the limits: a possibility-tension near divergence out-pulls every desire, so the federation slows before it - against a steepening pull.

The same architecture handles distributed-draw on any shared resource: shared infrastructure (a railway used by many scopes), shared skills (surgeons drawn on by multiple clinics), shared facilities (a lab used by chemistry, biology, materials research), shared atmospheric capacity (every emitting recipe across the federation). Each is one pooled budget, governed in stewardship, with one gap, whose possibility-tension out-pulls desire before its limit.

5.4. Possibility-tensions and their fragility profiles

The ecological case generalises: **every** limit the federation runs against is a possibility-tension, and its **fragility profile** is the shape of how it steepens toward its tipping point. Wherever the federation maintains a system whose state can degrade gracefully across a range and then collapse qualitatively past a threshold, that system carries such a profile: a continuous curve mapping state to **pull** in labour-hours, calibrated to the system's own dynamics. The mathematical form is shared - a smooth tension that diverges as state approaches the threshold - but the **content** of the curve (state variable, tipping point, steepness, asymmetry) is calibrated per system, by the stewardship scope that knows it best. Among the limits a profile prices is the **insurance** fund itself: the same registered curve that prices a live limit also sets, per (category, resource), the insurance-rate ι_r - how large a calamity reserve to hold behind the good, and how steeply to defend it as it empties. Sizing the insurance fund and pricing a fragility are one governance act.[27] The soft **buffer** (b_r, *Desire on the turn*) is the separate, **wanted** thing - sized by ordinary variability, traded in desire - and never touches this machinery.

A profile may defend a band, not just a floor. Some limits are **two-sided**: a nutrient has a deficiency floor *and* a toxicity ceiling, a reservoir a drought minimum *and* a flood line, a buffered

[27]The rate is a governed row: `module/playnet/planner/knowledge/catalogues/insurance.scm` carries, per resource category, $(\iota, \text{margin}, \alpha, w)$ - the twin of the conformance catalogue that governs the inspection barrier - and a live scope stands the reserve $q^{\text{res}} = \iota \cdot q^{*}$ behind every good it wants by default (`plan-scope #:insure?`, on unless a scope opts out — a ratified row is a decision already made), as a `held` floor with a margin in the good's reproduction band, taken off the on-hand so the want counts above it; the plan reports the cell's tab (`reserves`: desired, reserve, target, level). `tests/planner/test-insurance-live.scm`.

good a stockout *and* a spoilage bound. The federation defends the whole **safe band** with the same barrier, read off the **nearest wall** - $s = \min(s_{\text{lo}}, s_{\text{hi}})$, the normalised slack to whichever edge is closer - so the possibility-tension $\varphi(s) = -w \log s$ diverges at **either** end and is flat and free across the interior: *not too much, not too little*, and anywhere safely in-band costs nothing. (Equivalently it is the max of the two one-sided barriers.) Crucially this **does not leave the one convex program** (§The program, exactly): across the band the slack $s(x)$ is **concave** - it climbs to a safe plateau and falls away past each wall, a min of affine pieces - and $-\log$ is convex and decreasing, so $\varphi(x) = -w \log s(x)$ is **convex**, a convex-nonincreasing function of a concave one. A one-sided profile - skill redundancy, an ecological envelope, the burnout ratio L/A - is simply this band with its far edge at infinity. So two-sidedness costs the solver nothing: the harmony program stays convex, and the **reproduction of labour-power** (§The categories of social reproduction) is defended almost entirely by such bands. In the program the band is carried as its two one-sided terms on the one good - a floor at the deficiency edge and a barrier at the toxicity edge - which is the same convex object as $-w \log \min(s_{\text{lo}}, s_{\text{hi}})$ (a max of two convex terms is convex, and a sum of them is a smooth majorant that diverges with it at either wall) without the kink the min puts at the crossover, so the Newton steps that solve the program have a smooth φ' and φ'' everywhere in the band. Both walls are one kind of object - a wall, its range of uncertainty $[lo, hi]$, and the point at $\{\text{quantile}\}_{\alpha}$ inside it that the plan defends - but the **shape** of the tension at each wall follows from its **side**, and this is forced, not chosen. At a ceiling, thrift and the barrier push the same way - draw less - so the log barrier $-w \log s$ settles: the defended point is where the plan may **never go**, and it is felt before it is reached. At a floor they oppose - thrift pushes the level down, the barrier up - and the same log barrier meets thrift wherever the weights say, at wall $+w/\kappa$: a number about weights, not about need, over-making by an order of magnitude as labour cheapens. So a floor is held by the **held** tension instead - divergent at bare need, so nothing crosses it, and **satiating** at the defended point, so nothing is over-made for it: the point on a floor is where the plan is **at ease**. One object, two promises; a band is an **assembly** of the kinds the program already has, and a kind is only ever added for a curve no assembly of the others can draw.[28][29]

What a person will not eat is a fact about the person, like age - part of what the program is built from, never a term it optimises over. The floors above are computed from attributes and closed by the **eat** recipe the plan builds for every (person, food) pair; a diet is a statement about which of those pairs exist. A food carries what it **is** as classifications on its specification - `vf:food-class:dairy`, `vf:allergen:contains-milk` - and a person carries what they **refuse** as typed tags beside their age and sex - `diet:vegan`, `allergy:nuts` - and one predicate, *eats?*, builds the recipe or does not. The floor stays hard and untouched; the plan closes it from the foods the person accepts, and where the diet takes a nutrient off the menu entirely - no food they eat carries B12 - the floor for it cannot stand and the gap is **said**, naming the person and the nutrient, never a

[28] Measured in `tests/planner/test-shape-by-side.scm`, each shape put on each side of one rig: a log-barrier floor moves more than tenfold across a fiftyfold sweep of the labour price while `held` sits on the tip within 0.2; a log-barrier ceiling never crosses the tip however loud the want, where a `held` ceiling lets the want buy its way into the uncertainty zone. `docs/planning/solve/reproduction-bands.md` carries the tables.

[29] Realized in `module/playnet/planner/planner.scm` as one **reproduction band** per located good (`*band*`, `build-located` "REPRODUCTION BAND"): the deficiency edge a hard floor (a deficit on the nutrient's on-hand held by its ≥ 0 balance), the toxicity edge a fixed-weight `frag` on the nutrient's delivered net, one steepness. `plan-from-records` `#:bands` takes the catalogue's own row (spec, rda, ul, w) whole - `#:repro-floors` and `#:repro-caps` are spellings of one edge each - and `plan-scope` hands every person's bands through from the one subsistence catalogue (`subsistence-bands`, `knowledge/subsistence-catalogue.scm`) so the dial and the plan agree. `tests/planner/test-nutrition-band.scm`: a taste for a salty meal that a floor alone lets run past the UL is held inside the band by the two edges together, felt before UL, and yields to the ceiling.

silent zero and never a dead solve. A diet also moves the floor it makes harder to fill: non-haem iron is absorbed less well, so a vegetarian's iron floor is 1.8 × and a vegan's zinc 1.5 ×, one more layer of the same catalogue, folded between life stage and labour. Two neighbours are **not** refusals and go elsewhere. A **preference** - lentils over beans - is a want on the person's own good and stays in the objective where trade-offs belong. A **partial constraint** - dairy at most twice a week - is a ceiling, and convex: a barrier on the person's own eat columns, the toxicity edge's shape applied to an act, felt per period and per person, never a ration on the food for everyone. Only refusal is structural.[30]

The eat recipe is composition read off the recipe (§Material flows), pointed at a person. The one closure that carries a crop's element masses into the soil's ledger carries a food's nutrients into the body's band; a food carries both at once - the rift leaving the loop and the subsistence entering the person - and neither wall asks which food delivered it.

A partial catalogue of the possibility-tensions the federation tracks this way - each one a fragility as it nears its limit, and each stated over a **substance** - a nutrient, an element's mass, a condition, a ratio of hours - reached from whichever forms carry it. What varies down the list is the substance; the object is one (§Worker, soil, and tool: one field):

Ecological agents. Forests, fisheries, soils, watersheds, the atmosphere - state variables are population, biomass, fertility, concentration; tipping points are regime shifts, collapse thresholds, irreversibility horizons. Stewardship at the agent's custodian scope.

Skill redundancy. For each skill k, the number of trained practitioners approaches a critical floor. One remaining surgeon is fragile; two is slightly less; ten is robust. The fragility profile rises sharply as redundancy approaches one, especially for skills with long formation lead times where loss cannot be quickly replaced. The federation feels this as rising allocation cost on any commitment that consumes the practitioner's time. This is the Skills category's living insurance margin (§The categories of social reproduction) read as tension - one decision, sized once. It is also the one wall in this list stated on a form: a **count of people**, with no substance beneath it, because a skill enters no good (§Reproduction labour and the two measures of labour).

Labour pool burnout. The ratio L/A measures how much of total offered availability the federation is currently consuming. Far from one, citizens have slack and the federation has surge capacity. Approaching one, every additional commitment risks individual exhaustion and reduces the federation's resilience to disruption. The fragility profile on L/A rises long before saturation, expressing the federation's care for its own people as allocation cost. It is the one wall on hours at its coarsest grain: the located balance per skill and cell, and the EMP congestion per worker (§Labour is a good on the lattice), are the same limit read finer.

Care capacity. The ratio of available caregivers to those in care (children, the sick, the elderly, the bereaved) carries its own fragility. Past a threshold, care quality degrades sharply: relationships become transactional, attention becomes thin, the cared-for absorb the burden of their

[30] Realized in `module/playnet/planner/knowledge/catalogues/diets.scm` (`eats?`, `diet-exclusions`, `food-classes-of`), read by `scope-consumption` (`live.scm`) per person, with the floor rows as the catalogue's own `*diet-shifts*` table (`subsistence-L2-diet`) mirrored in the dial's `nutrition.ts`. A person states all of it - life stage, sex, age, climate, labour, diet, allergies - in one settings section, written as typed tags onto their own agent record (`set-profile`); the floors are provisioned to that description, and nobody's plate is counted. `tests/planner/test-diet.scm`: a vegetarian has an eat recipe for beans and none for fish, her protein floor stands, her iron floor reads 1.8 × her neighbour's; a vegan on the same menu loses her B12 floor by name.

own care. The fragility profile here is what defends the federation against the trap every prior society has fallen into - of extracting reproductive labour past its sustainable rate.

Energy and infrastructure. Grid load approaching capacity, reservoir levels approaching minimum, bridge loadings approaching design limits, server capacity approaching saturation. Each engineering system has a fragility profile rooted in its physics: damage probability rises non-linearly as state approaches the limit, and the profile makes that physics visible in allocation arithmetic.

Inter-scope dependence. When a scope's input depends entirely on a single upstream supplier, the dependence is fragile - one disruption breaks the chain. The fragility profile penalises monoculture in the supply graph and rewards recipe diversity. Multi-supplier resilience is a continuously enforced structural property.

Demographic reproduction. Fertility rates, age distributions, population replacement. A society whose demographic state drifts toward unsustainability has a fragility profile that should rise long before the demographic curve becomes irreversible. This is among the deepest fragilities and the one most economies have historically been blindest to.

Knowledge transmission. When critical tacit knowledge is held by a small number of ageing practitioners and not being transferred to the next generation, the knowledge stock approaches a tipping point of irretrievable loss. The fragility profile here pushes the federation to invest in apprenticeship, documentation, and intergenerational transfer before the knowledge becomes unrecoverable.

Biodiversity within managed systems. The genetic diversity of food crops, livestock breeds, seed banks, microbial cultures. As diversity drops, the system becomes monocultural and vulnerable to single-point failure (a single pathogen, a single climatic shift). The fragility profile rises as diversity approaches genetic uniformity.

Social cohesion. Trust levels, conflict-resolution capacity, mediation availability, the felt sense of solidarity. Hard to measure but real - and degrades gracefully before any catastrophic break. The federation can carry indicators (recognition rates, mediation case loads, intergenerational interaction levels) whose fragility profiles flag when the social fabric is thinning.

The general principle: **the federation maintains continuous awareness of all its possibility-tensions - all its fragilities - in the same labour-hours it uses for everything else.** A desire-satisfying allocation that pushes a system toward its tipping point reads, in harmony, as a steeply rising **pull** - and harmony finds a different allocation that resolves less desire but eases the fragility. The trade-off is **visible, continuous, and structural**, not a hidden externality.

The catalogue is governance-set, not engineering-set. Which fragilities the federation tracks, which thresholds it accepts as tipping points, how steeply the curves rise - all of these are political-economic choices, made by stewardship scopes for the systems they speak for and ratified at the UC level.[31] A federation that runs its forests close to tipping has chosen aggressive forestry; one

[31]Implemented as a governance object, not a hard-coded table. The fragility catalogue is a registry actor: registering a kind is a production-governance act, and the solver holds only an attenuated, read-only view of it. From a single specification - for each kind its value $\varphi(s)$, its gradient $\varphi'(s)$, and its curvature $\varphi''(s)$ - the registry compiles the planner's inline per-kind term math, so a new fragility profile enters as one datum rather than as parallel edits across the solver's hot loops and the compiled C kernel, both of which are generated from the same spec. Each kind's analytic gradient and curvature are checked against its value function by finite differences, and proved outright in the certificate: `SolverCertificate/TensionKinds.lean` derives every kind's gradient and curvature columns as

that runs them far from tipping has chosen conservative stewardship. The architecture makes the choice visible and inspectable.

A limit is also governed in its SHAPE, and there are two. Some envelopes **renew**: a season's streamflow returns next season, a fishery not fished past collapse spawns again, an aquifer recharges. Their limit is a **rate** - what may be drawn per period, and the headroom resets when the period turns. Others do not renew on any horizon a plan can see: old growth, a drained fossil aquifer, an extirpated run. Their limit is a **total** - spend it and it is spent, and no later period hands any of it back. The distinction is not a modelling detail but a claim about the world, and so it is governed alongside the threshold and the curve: to inscribe an envelope is to say both where its wall stands and whether crossing toward it is undone by time.

Getting it wrong is silent in exactly one direction. An irreversible limit written as a rate refills every period, so a plan may cut an old-growth stand in each of thirty years and never once read as having pressed a limit at all - the barrier was satisfied thirty separate times. The reverse error is loud: a renewable limit written as a total simply strangles a scope that could have gone on drawing forever.[32]

What distinguishes this architecture from any predecessor: **the federation cannot be fragility-blind.** Capitalism is structurally fragility-blind - externalities don't appear in price signals, and the system discovers fragility only by collapsing into it (financial crisis, ecological breakdown, pandemic, burnout). State socialism was partially aware but its accounting was discrete and reactive (five-year plans, crisis-mode interventions). The federation expresses every fragility in the same continuous currency as every other planning decision, and confronts it on every event, in every commitment, at every cell of the surface. Fragility becomes ordinary accounting rather than emergency response.

This is what makes the federation **prudent by construction**. Not because the people in it are wiser - they may not be - but because the architecture refuses to let them be blind. Every fragility the federation has inscribed is one it cannot accidentally cross. The remaining risks are the fragilities it has **not** inscribed: the natural systems it does not yet recognise as agents, the thresholds it has misplaced, the curves it has set too gently. Those are the open work of governance, and they are the federation's deepest political-economic responsibility.

5.5. Worker, soil, and tool: one field

The catalogue lists them apart - the labour pool's burnout ratio, the ecological agent's regenerative envelope, the engineered system nearing its design limit - but read together they are **one form**. A body held above its nutrient floors, a soil held below its regenerative envelope, a tool held above its breakdown condition: each is the **same** convex possibility-tension - a wall, its range, and the point in it the plan defends - read at a different scale, the log barrier $\varphi(s) = -w \log s$ where the wall is a ceiling on a draw and its floor-sided twin, the held tension, where the wall is a level to keep

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the true first and second derivatives of its value, so a sign or factor error is a build failure. See `module/playnet/planner/formulas/tension-kinds.scm` and `tension-registry.scm`.

[32] Realized in `module/playnet/planner/solve/engine/fragility.scm`: a fragility names its **extent** (which draws it measures - one located good, a spec wherever drawn, an ecological container's contents, or everything at or under a place) and its **window** (a rate per period, or a total across the horizon). The two are orthogonal, and the same profile (lo, hi, α, w) is read at whichever scale the pair names - which is what §"Worker, soil, and tool: one field" means by one form read at a different scale. `test-fragility-scales.scm` puts one draw under each scale and reads what binds.

(§Possibility-tensions and their fragility profiles). So the plan defends **three** reproductions in one solve - of labour-power (the body's floors), of the land (the soil's envelope), and of the means (the tool's condition) - and the worker, the soil, and the tool are kept **together**, none run to exhaustion for the sake of the product. The subjective and objective conditions of production wear alike, and are held above their walls by one barrier - the depreciation each means sheds into its product (§Labour, skills, and the surface) and the fragility each nears as it wears are the **same** wearing, read as value and as tension.

Stated once, for every wall the catalogue will ever hold: **every wall is written on a substance.** A floor or a ceiling is written on what the constraint actually concerns - a nutrient, an element's mass, a condition, hours - and reached from the concrete goods that carry it by a closure blind to which of them supplied it: the eat recipe for a body, the composition read off a recipe for the soil, the condition stock a use draws down for a tool, the value rollup for the pool. Desire - this food, this crop, this tool reached for - stays on the form side, in the objective where trade-offs belong. A wall written on a form is either too strict, refusing an adequate substitute, or too loose, missing one the catalogue happens not to name; a wall on the substance is neither, and needs only the closure that finds the forms kept current. The companion note *Form and Substance* (`docs/form-and-substance.typ`) traces this one reduction from field to body and to Marx's own use of the pair.

5.6. The condition is a stock, mined or tended

Read in one solve, such a condition is a barrier; read across the **periods**, it is a **stock that carries**. The soil a crop draws this period is the soil the next inherits, so the condition obeys a recurrence - condition next = condition now − draw (production) + restore (regeneration), never below its floor - threaded period to period by the **same storable carry** (§The surface as a stock network) that banks a granary against a drought. This is no new machine: it is the stock balance the plan already keeps, read down the **time** axis. Soil fertility, a tool's serviceable life, a worker's competence are each a level the producers may **mine** - spend now for more product, and stand poorer next period - or **tend** - spend labour now to hold it, and stand as rich again.

The payoff for tending is the **arithmetic of the recurrence**, and it **emerges** — no rule states it. Set a subsistence floor on **every** period and place a heavy-feeding staple beside a soil-restoring legume - a crop whose growing **fixes** more nitrogen than it draws - and the plan that must keep feeding **discovers crop rotation**: it mixes in the legume, lower-yielding but soil-building, because that is the cheapest way to stay feasible across deep time. A monoculture that mines the topsoil feeds the near periods and **spends its soil-life** - it is on borrowed time, and the plan can read the period it comes due; the rotation **banks** the soil-life instead. The same recurrence carries a forest - standing timber drawn by logging, rebuilt by bounded regrowth, where a regrowth-fed cut outlasts a clearcut that starves the later periods. So the metabolic rift - a nutrient washed out and not returned, serviceable life leaving the loop for the scrap-heap - is not a moral the plan is handed but a **cost it internalizes**: the soil kept whole is the **cheaper** plan once the floor is due every year, not only the better one.[33]

[33]Realized as a general *regenerating stock* (`module/playnet/planner/formulas/stocks.scm`): a condition is a storable good with a starting level, drawn by a `consume` flow on the production recipe and restored by a `produce` flow on the regeneration recipe - the durable-condition pattern lifted to a first-class, reusable declaration that fits soil, tool, pasture, aquifer, or fishery alike. Worked on the real Doon-Valley farm (topsoil at the measured SoilGrids state, soil organic carbon 34.3 $g \cdot kg^{-1}$), where a rice-vs-pulse rotation emerges to hold it above the degradation floor while a monoculture mines toward it (`test-soil-recurrence.scm`), and on a *forest* (ESA WorldCover cover index) where a regrowth-fed cut outlasts a clearcut (`test-forest-recurrence.scm`). The per-crop and per-stand draw/

5.7. Nothing enters free: the last externality

The catalogue closes a system's **known** walls; but there is a prior question - how does anything enter the account at all? A good may enter in only a few **honest** ways. It may be produced - a recipe makes it, its labour and matter on the books. It may be a stock - a standing level drawn and regenerated, carried through time. Or its draw may be barrier-bounded - a possibility-tension pricing the approach to a tip it never crosses. What is left over is the **free raw**: a good drawn with **no producer, no stock, no barrier** - no labour, no limit, no account. That residue is the externality in its exact form - the free gift of nature, appropriated and never entered - and it **is** the metabolic rift at its root.

So the plan's default is **fail-closed**: a draw the model has not accounted does not quietly conjure itself; it **dies**, as loudly as a missing recipe, until it is entered as one of the honest three. The rift is not patched draw by draw - it is **closed by construction**. Only what is genuinely boundless - sunlight, rainfall, what no reproduction bounds - may be a free raw, and only where **declared** one in the open, surfaced so that every remaining externality is a conscious choice and never a silent default.[34] This is the same move against the rift as the catalogue makes against the tipping point - refuse the free lunch by default, and let every real cost find its place in the one ledger. To produce as **associated producers, rationally regulating their interchange with nature** is not to draw upon nature freely, but to leave **no draw upon it unaccounted**.

5.8. Reproduction under uncertainty: the floor on every path

The world a plan reproduces its means into is not one world but a **distribution** of them - a good monsoon, a drought, a flood, and the failure worse than any of them. The reproduction floor - everyone fed, the soil held, the tool kept - must hold not on **average** but in **each**. This is not pedantic: a society lives **one path** through time, and want at the floor is **absorbing** - a season of famine ends the path it strikes. So an average over worlds that never both happen governs nothing here. The plan keeps the hard/soft seam it already has (§The categories of social reproduction): the floor is a possibility-tension in **every** scenario, unweighted by probability, and the surplus is judged scenario by scenario - each future's captured share of its own achievable value, the plan lifting the least of them, with no probability among the weights (the maximin reading, developed below; an expected-value weighting by the world's likelihood is the engine's alternative, off in live solves). Robust floor, robust surplus. The record's probabilities place the reserve instead: it stands in every recorded future and yields only in the $p = 0$ tail.

Nor does the world resolve in one instant. A growing season's weather is not a single guess made at planting and settled by harvest, but a sequence of degrees of knowing - dry so far, and now also dry, or now wet after all - and what to plant at the **second** season may honestly depend on what the first one delivered. It may never depend on what it has **not yet** delivered: two farms that lived through the identical dry first season must reach the identical second-season decision,

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restore coefficients are first-pass, ranked by agronomy (FAO/IPNI nutrient removal); the mechanism is exact, the numbers honest placeholders.

[34] One predicate carries it (`spec-skips-balance?` in `module/playnet/planner/planner.scm`): a good skips its ≥ 0 stock balance only for a planner-generated structural pool, a *barrier-bounded* draw whose fragility form *is* its balance (`barrier-bounded?`), or a spec listed in the boundless-ambient allow-list `*raws*`. Everything else must be produced or stocked. `PLANNER_RAW_LINT` prints the STARVED specs (consumed with no producer, stock, or barrier anywhere), the STRANDED ones (produced somewhere, drawn where no carry reaches, so the prune removes that draw before the solve - a dead column rather than a hole), and the EXTERNALITIES actually drawn as free raws - confirm each is truly boundless, else give it a stock or a fragility.

whatever they each privately fear the third might bring, because nothing has yet distinguished them and a plan is not a guess about which of them will get lucky. The floor keeps this the same way it keeps everything else - not by writing down an equality between two guesses and hoping the solve respects it, but by handing every scenario that has not yet diverged the **same** column to fill. Two numbers are identical because they are, arithmetically, **one** number; nothing was ever in a position to make them disagree.[35]

The probabilities come from the record - but the record **undersamples the tail**. The $\pm 1\sigma$ that sorts years into normal, dry, and wet assumes a thin Gaussian bell that rainfall does not have; the worst monsoon that **will** come is not in the sixty years measured. So beyond the observed scenarios the plan carries a **precautionary tail** - a famine-scale failure at **probability zero**, making no forecast, only refusing ruin. Because probability weights only the surplus, a zero-probability scenario adds nothing to what the plan **pursues** - yet its floor is still defended. Survival is guaranteed against a failure about which the plan claims nothing.

And the probabilities themselves are **derived numbers with a provenance**, never typed into a configuration. The drought/normal/flood split the Doon Valley solve reads - $(0.14, 0.67, 0.19)$ - is the classified 1960–2023 ERA5 rainfall record of that specific watershed: checked-in readers turn the raw series into the split (the IMD $\pm 1\sigma$ classification) and each branch's shortfall into its yield multiplier (the FAO Ky water-deficit model), and their output is committed as **generated, do-not-edit** code behind a SHA256 gate on the vendored raw data - a hand nudging a probability shows as a build diff against the record. Any latitude, any bioregion is one command away; the method generalises past the valley it was proved on. This is the discipline that **licenses** statistics anywhere in the plan: a domain earns its confidence exactly where its generating process is shown this concretely, and everywhere the grounding is thinner the plan reads the barrier's bare slack, probability-free by design.[36]

A figure's **witnessed** failure is a third source of shock, beside the record and the tail. A worker of a part whose own work event read a yield below the recipe's, or an input or hours above it, signs a **contrary observation** (*The voice field*, "Feasibility follows the map"), and the solve reads the figure at that reading in the cautious direction alone - a produce at most the nominal, a draw or hours at least it - laid over every scenario, the record's and the tail's alike. It is the same calibration a realized shortfall already applies to a shock, taken from one witness instead of a season: constative, guarded by the witness event and by audit, standing until the map is corrected.[37]

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[35] A scenario carries an optional `path` - the branch labels it has followed - and two scenarios share a period's column exactly when their paths still agree there (`scenario-node`, `module/playnet/planner/solve/engine/robust.scm`), proved against a genuine two-branch-point tree in `test-scenario-tree.scm`. A weather shock can likewise name the one period it falls in rather than every period its process runs, so one recipe carries a different draw at each season instead of needing a copy per season (`test-period-aware-shocks.scm`). Both fall back to the flat, single-branch case byte-for-byte when no scenario in a set carries a path or a shock names no period - the sequential case is the general form, not a rewrite of the one this section already described. `docs/planning/solve/scenario-tree.md`.

[36] `module/playnet/planner/knowledge/data/`: `bioclim.scm` (rainfall → scenario probabilities), `agronomy.scm` (FAO Ky + OWID yields), `emit-derived.scm` (the generator producing `worlds/doon-derived.scm`), `process*.scm` (Guile-native readers for ESA WorldCover COG and RESOLVE ecoregions, feeding the fragility state variables directly - no third-party geo stack), `fetch-*.sh` (SHA256SUMS-verified), `sources.md` (the full raw → derived → checked-in chain).

[37] `observed-shocks` and `scenario-with-shocks` in `module/playnet/planner/solve/engine/robust.scm`, fed by `contrary-observations-of` off the governance store through `plan-scope`'s `observed` keyword; the credential is `playnet-contrary-observation/v1`. `tests/planner/test-contrary-observation.scm`: a half-yield oven roughly doubles the bake hours in every future; a reading above the nominal changes nothing.

What is not known is not given a number. Three readings the festival's live solve settled the same night (`docs/planning/solve/antifragile-sweep.md`):

- **A permission is a wall, not a future.** That any one district may withdraw its pits after planting was first stated as twelve futures, one per district, at odds nobody had (0.30/12 each under a 0.70 hold), and forty branches to carry them. The right states it as a **budgeted floor** (Bertsimas–Sim): a right held `against:`Γ, met under the loss of **any** Γ of the places that supply it, $\sum_d y_d - \Gamma z - \sum_d p_d \geq F, z + p_d \geq y_d$, laid in the good machinery as synthetic balances and zero-labour columns, its dual the price of the budget. A **source** is a place with supply of its own - a producer there by the floor's period, or stock standing there - so what an origin merely passes on is not counted, and y_d is bounded by what d itself produces, never by gross inflow, or a harvest circulated through a place would read as two. Berlin's day: 40 futures in 64 s became 4 in 3 s, the carries spread over every district.
- **The wants are judged without a probability.** A want's captured value in a future is its satisfaction - delivered over asked; a future's **available** value is the most its own program captures, planted for itself alone. The plan maximizes τ with every future capturing at least τ of its own: the epigraph of a min of linear functions, convex, `maximin-extend`. The wants' own terms stay at one equal weight per future - the tie-break among plans with the same τ - and the record's odds steer nothing; the $p = 0$ tail counts like any future. A want a future cannot serve at all enters the wall as a constant, as a want no column feeds already does. The report carries τ and each future's fraction; Berlin's crowd reads $\tau = 1$ in every weather.
- **A relocation is recourse.** What is committed before the weather is what **transforms** a good; a process whose flows are a pickup/dropoff pair - a haul, a hold - is decided once the harvest is known. Committing the carries first-stage forced a carry to zero in every future once its origin's harvest was lost in one, and with every district lost somewhere no carry to the ground survived: a floor every branch alone met read unmeetable, the whole floor, linearly.

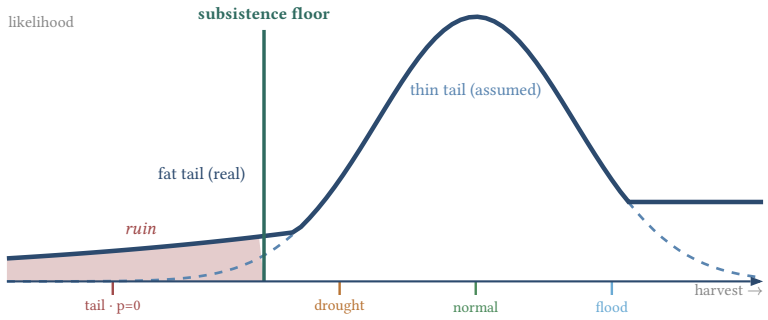

Figure 7: **The tail the record hides.** The historical years cluster under a thin bell and are sorted $\pm 1\sigma$ into drought, normal, flood. But rainfall is fat-tailed: harvests far below anything measured carry real, unestimated probability, and below the subsistence floor lies ruin - absorbing, not to be averaged away. So the plan adds a precautionary tail at $p = 0$: it makes no forecast (weighting only surplus, a zero adds nothing to what is pursued) yet its floor is still defended.

This reshapes the most basic question a bounded means can be asked: how many can it **guarantee**? The record says one number; the tail says a smaller one. A two-hectare Doon Valley smallholding "feeds" twenty-eight against the historical distribution - and delivers barely a third of their food in the tail; the **ruin-robust** number, guaranteed through a failure beyond any record, is fifteen. The record's carrying capacity is an **over-promise**. And here the account closes on itself: robustness **survives** shocks, *antifragility* **gains** from them. A good year's surplus is not waste to sell off but the reserve that carries the tail - a granary banked from good years **raises** the guaranteed capacity,

the twenty-eight the record over-promised made survivable once a fraction of a good year's surplus is held back. It is nothing new: a reserve is the reproduction floor's own mechanism - a starting on-hand stock - run the other way, and a plan over a multi-year horizon with the floor due each year over-grows and rations forward **of its own accord**. And it does so **before it knows the weather**: with the futures laid out as a tree - this season known, the next one of the record's monsoons or the tail - growing and storing forward at the root is one decision shared by every branch, and the insurance reserve q_r^{res} on next season's food, held in every future, makes the plan bank exactly what the worst defended future needs, against a market that wants the surplus now - a granary sized by the computation of probabilities, where a plan for the **mean** monsoon banks nothing and lets the tail starve. The same split again: the reserve clears the tail **and** frees the land for the high-yield upside.[38]

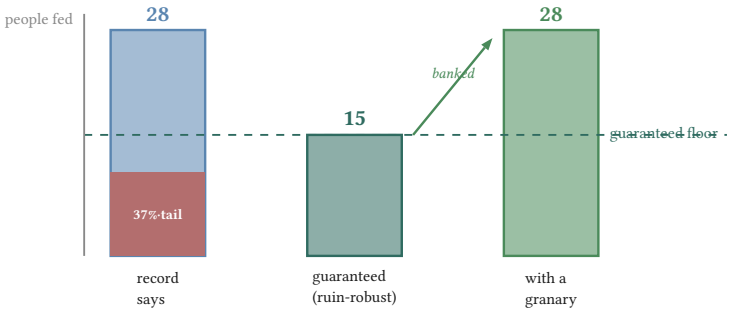

Figure 8: **Carrying capacity: the over-promise and the buffer.** Against the historical distribution the farm appears to feed 28 - but in the fat tail it delivers only 37% of their food. The number it can **guarantee** through a failure beyond the record is 15. Antifragility closes the gap: a granary banked from a single good year's surplus lifts the guaranteed capacity back to 28 - the good years' volatility stored as survival, not sold off. A reserve is the reproduction floor run in reverse; here it is what turns an over-promise into a guarantee.

5.8.1. The accept: a person decides a column

The tree's two kinds of decision are also the plan's **social** interface. A suggestion the solve emits is **recourse** - branch-private, re-cut freely on every re-solve, promised to no one. An **accepted commitment** is the discrete layer's third operation performed by a person: a column **decided** - a fact, substituted out of the program through the same door the branch-and-bound search fixes its own counts - plus a booked binding at deployment, so a committed person is never assigned twice. The pin enters after every scenario block, so bounds propagate: a pin that starves a reproduction floor in **any** future raises, refused rather than solved around. And pinning implies first-stage sharedness - a fact cannot differ per branch - so an accept carries non-anticipativity as a consequence of something stronger. What a person wants from standing ground - one value, identical in every future, plannable around no matter what happens - is precisely what a decided column

[38]The robust split (floor enforced in every scenario, the wants judged scenario by scenario without a probability, plus a precautionary $p = 0$ tail) and the exogenous granary are `module/playnet/planner/solve/engine/robust.scm` with `navdanya-farm`, `taleb-tail`, and `antifragile-reserve` tests; the carrying-capacity figures are their outputs on the real Doon-Valley calibration. The endogenous granary under uncertainty is `tests/planner/test-granary-under-uncertainty.scm`: the same farm as a scenario tree with `#:insurance` on next season's food (`held`, kept per scenario) banks 1 825 kg of a 5 420 kg harvest, sized to the famine tail; the certainty-equivalent world and the soft buffer bank nothing. Every wall - `bar`, `frag`, `held` - is kept in every scenario; only wants are probability-weighted. The same $p = 0$ tail is where needs rest on what has been proven: there a recipe yields by how far its record reaches beside the best-covered recipe for its output, so the floor is held by what has met such weather (§Character is a measurement).

is: the human comfort and the solver object coincide, and the mechanism honors the feeling while pricing its cost honestly, the classical gap between deciding now and deciding when the weather is known.

Between free and decided sits **bounded**, and it owns no storage of its own: **the hold is the open offer** - a directed proposal with an acceptance window, compiled to a wall $x \geq v$ that rides every intervening re-solve, so at accept time the value is certified by the latest solve alongside every standing pin. The window closes at the **forcing point** - the moment past which the work's own lead time can no longer meet its date - and accepting rides the ordinary accept door: the promotion finishes the intent, which is the hold's release, and the commitment pins the next solve. Both lapse doors are loud and land once each on the alert rail: `offer-lapsed` when the window closes unaccepted, `offer-infeasible` when the held program has no answer but the free one does - the solve retries, names every holding offer, and the free program stands. The decided rung has its withdrawal too: a release, provider-owned, finishes the commitment where it stands - the record stays, and the next solve drops the pin exactly where the accept was felt.

How far ahead an accept may reach is arithmetic, never a policy constant. Both of the gate's numbers come from one algorithm read twice: augment the work's recipe DAG with each input's booking notice as an edge - read off the input's **own** availability windows, MIN over alternative suppliers, the same combine the federation fold uses - and run the critical-path pass. The **ceiling** is the forward number, maxed over the branches the commitment must serve: a want is served in expectation and reads its one chain; a floor is defended in every branch, so the worst defended branch's longer chain **is** the floor's extra reach - the margin derived. The **urgency** is the same pass read backward: the least slack across those branches, or the offer's own closing window, whichever comes first. Urgency never widens a ceiling - high slack on a fungible resource reads as zero urgency **and** a short ceiling, so locking in months ahead for peace of mind is refused by arithmetic: the stability that request is after is the **standing declaration**, which costs no recourse at all. A Monday-only input read on a Thursday legitimately widens the reach by four days; a second, always-open supplier collapses it again.[39]

5.9. Character is a measurement: the recipe under stress

The reserve defends the **floor** against a bad future; a recipe's own conduct under stress is read the same way, so that what commands the common means is exactly what has earned it. A recipe enters as a **cited hypothesis**. Each run is scored against what the realized weather **allowed** - a harvest of four-of-ten under a drought that permitted four is a full capture, a practice answerable for its conduct in the world it got and never for the world - and the weather itself is read, never declared, off the same witnessed events. What a run draws and what it makes are two readings: its inputs arriving short - hours, seed, a means it uses - are weather it worked under, and its output

[39] `robust-assemble` `#:pins` and `#:holds` (`module/playnet/planner/solve/engine/robust.scm`) enter through the discrete layer's own `substitute-fixed`, and a pin joins each search node's fixed-column table, a fact in every node of the tree. `accepts.scm` carries the two doors `plan-scope` crosses - `commitments-before-solve` (expiries, accepts → pins, open offers → holds) and `commitments-after-solve` (the horizon read cached, unbound floors named, freed holds lapsed) - plus `deployment-bindings` compiling standing accepts to `#:seed` and held offers to `#:tentative` for every matching round. `offers.scm` holds the people's doors (`offer-work`, `present-offers`, `accept-regret`, `commitment-drift`); `horizon-gate.scm` the gate (`augmented-ceiling-days`, `booking-notice-days`), enforced at both accept doors - the calendar's promote and the proposal's accept - and re-armed on every persistence reload by the scope's `rewire-callbacks`. Every ending and lapse is a dated fact on the plan-store's rail, one shape, one writer; `docs/planning/solve/commitment-horizon.md` is the arc, and five suites over `tests/bread.scm` prove it by concern.

against what that weather allowed is what it captured, the scarcest input setting what it could have made. So a recipe alone in its scope still has a sky: what reached it. On its output the sky is the best run of that day, whoever ran it. Stress and allowance are two readings of one record and describe the one world a run lived in. What the runs trace is a **curve**, per flow of the recipe's own - its output, each input, the skill it works: how much of the achievable the recipe captured at each stress the weather delivered there. Stress is one reading of one function: what a list of shocks does to the recipe's own flows, by the matcher the solve itself applies, so the sky a run lived under and the future a solve plans against are read alike and a record meets every future that touches the recipe, however its shocks are keyed. A run is filed under the flow its weather pressed hardest, a calm run under each at no stress; a flow the weather has yet to press reads, the first time a future presses it, the most adverse response the recipe has shown at that stress - ignorance reads as the dangerous case - and its first run under that weather gives it a curve of its own. The curve is **read, never stored** - one reading per stress level observed, in proportion between two levels, the lightest reading held below the record, and beyond it harm continued at the last observed rate while a gain holds where it stood.[40]

The plan responds in proportion. In every future of a solve, a recipe yields what its own curve says at that future's stress: one more shock on its own acts, composed with the weather's by the one matcher every shock goes through. Floors are held in every future (§Reproduction under uncertainty), so the plan leans on a recipe future by future, by what the recipe delivers there - less on what breaks under drought, more on what holds where others fail - and the lean moves as the record moves. Each future is read on its own and never at the mean of them, so a response that bends is felt as it bends. Two ways to grow grain from one store of seed, the quicker making half again as much per seed: with no record the plan plants the quicker alone; once its record shows half of what a drought allows, three parts in four of the seed move to the steadier; at three parts in ten, nearly nine in ten move - and the quicker still runs where it pays.[41]

Which reading a stress level takes, where several runs witnessed it, is the regime's. Statistics is licensed here the way the scenario probabilities are: where a live, unrevoked credential asserts the domain has thin tails **and** a stable process, a level reads at the **mean** of its witnesses, and the fitted acceleration rides beside the curve with its standard error - capture is a bounded share, where a mean is a sound reading and every run counts in it. Absent, lapsed, or claiming anything else - honest ignorance included - a level reads at the **most adverse** capture witnessed, `probability-free` by default, because a small record hides its bad runs and misreading a fat-tailed domain as a thin one is the asymmetric, dangerous error.

The triad is a description of the curve, read off the three stress levels that bend the most: the gap, in points of capture, between the middle reading and the straight line joining the outer two. Harm that **accelerates** - capture falling over the heavier step, faster than over the lighter - is **fragile**; a

[40] `module/playnet/planner/solve/recipe-response.scm` - the pure half: `response-curve`, `capture-at`, and `response-shocks`, which lays a record over a future. `module/playnet/governance/recipe-standing.scm` is its live adapter: `observer-runs` assembles a run from the Observer's fulfilling events, with what the weather allowed of a flow and how hard it pressed on it read as one figure (`flows-factor`, `module/playnet/planner/formulas/shocks.scm`) off the **same** scenario-shock record `engine/robust.scm` solves against, and `scope-standing` hands out one record per recipe - its acts, flows and curves for the solve, its rows, yield and runs for the face. `docs/planning/solve/recipe-metabolism-barbell.md` is the design note.

[41] `plan-scope #:responses` (`module/playnet/planner/scope/live.scm`) lays `response-shocks` over each future beside the contrary observations; the matcher is `branch-flow-factor` (`module/playnet/planner/formulas/shocks.scm`). Measured in `tests/planner/test-response-solve.scm`: 4.00 of the quick act with no record, 1.00 at a record of 50% under drought, 0.45 at 30%, the floor held in the drought each time. `tests/governance/test-recipe-standing.scm` reads the curves off a real scope's stores.

gain that accelerates is **antifragile**; a curve that holds across the stress observed is **robust**, and one that moves with the stress in a line is **in proportion**. Fewer than three stress levels - **awaiting observation**, visible and never guessed; a recipe is **observed** once some flow of its own has a run under stress, and **experimental** while every run was calm. Per flow, so one recipe may hold through a shortage of hands while it breaks under a failed harvest. The words are shown; the solve reads the curve.

The shape of common use is a **barbell**: nearly all of the common means run practices whose record is long, recertified as they are relied on, while a deliberate margin runs trials of what is not yet known - and standing moves by observation in both directions, a slipping practice leaned on less as its record shows it delivering less, a hardened one leaned on more. A record is a proof **up to the heaviest stress it has met**, flow by flow - its **reach** - as a crane is rated to the load it was tested at: a thousand calm days say nothing of the storm, and a run can rule a claim out and never in. So in the future a floor is defended in without a forecast, a recipe yields by how far its record **covers** that future - its reach on each flow pressed, over the stress delivered there - measured against the best-covered recipe for the same output. The furthest-reaching stands as its curve reads; a shorter record by its share of that reach; one that has met no such weather yields nothing beside one that has. Floors hold in that future, so every need rests on what has been proven; the future carries no weight in the expectation, so a recipe runs wherever it adds, and the most a trial costs is the surplus it was planted for: downside bounded, upside open. The bound is a construction and carries no number. Where no recipe for an output has been pressed, each stands as it is, the need rests on a record yet to be pressed, and the face says so. And a trial begins by consent: a person's offer to work a recipe holds its column, whether or not any plan chose it, so a practice the optimum would never plant earns its first runs - on means the floor can spare, since an offer that would take the floor's own is refused.[42] The loop then closes twice: observed fulfillment ratios become the next scenario forecasts, and the practices that proved themselves become the next plan's coefficients - **so the optimum itself climbs**, each solve re-run on inputs the last shock corrected. Markets also run a discovery loop - but their selection mechanism is bankruptcy, and the ruin lands on people, unbounded. Here the same evolutionary search runs with the floors outside every trial, the trials begun by consent and bounded by what has been proven: **discovery without ruin**. A shock teaches - which practices gained, which broke - and the lesson becomes allocation within a recertification cycle: volatility, which arrives whether anyone wants it or not, put to work for the commons at a bounded cost in means and none in lives.

5.10. Metabolic coherence

Social desire satisfaction (defined above) measures whether the output satisfies human desires. **Metabolic coherence** measures something structural: the ratio of a scope's internal flows to its total flows (where total flows equals internal flows plus boundary-crossing flows), between zero and one.

[42] `curve-reach`, `response-coverage`, `margin-shocks` and `futures-under-record` (`module/playnet/planner/solve/recipe-response.scm`), laid by `plan-scope`; the offer is `offer-work` (`module/playnet/planner/doors/offers.scm`), held by `standing-holds`. Measured in `test-response-solve.scm`, four of seed and a floor of 1.5: beside a tried act the never-run one takes 2.50 and the tried 1.50, which alone holds the floor; a recipe making less per seed runs 0.00 unasked and 1.00 on an offer of one run; an offer of 3.5 is refused. The construction is the barbell of `docs/antifragile/statistics.pdf` ch. 30 - the safe part covers the need "independent from all possible models" - and the reach is its figure 5.5, the turkey, read as a rule.

$$\underbrace{M(s)}_{\text{metabolic coherence}} = \frac{\overbrace{F_{\text{internal}(s)}}^{\text{internal flows}}}{\underbrace{F_{\text{total}(s)}}_{\text{total flows}}} \in [0,1], \quad F_{\text{total}} = F_{\text{internal}} + \underbrace{F_{\text{external}}}_{\text{boundary-crossing flows}}$$

Internal flows are those whose producer and consumer both lie within the scope. External flows cross the scope's boundary. Scopes compose, split, or merge to optimise metabolic coherence: high coherence means the scope's production-consumption cycle is **self-contained**; low coherence means most of its flows cross the scope boundary - it primarily serves or is served by other scopes. Social play time (the lower of free time and social desire satisfaction) measures the federation's joint progress toward free time and equal satisfaction; metabolic coherence measures the structural coherence of each branch of activity.

There are really **two coupled metabolisms** the federation must be coherent in:

Labour metabolism - denominated in crystallized SNLT. Every material flow carries the labour-hours that produced it; every purely temporal flow (care, instruction, repair, deliberation) **is** labour-hours directly. High coherence on this axis means the scope closes its own **human-time loop** - what its people produce, its people need; what they need, they produce.

Material metabolism - substance-denominated (kg of N, m^3 of water, MJ of energy, tCO_2e). This captures throughput **regardless of labour content** - including the unpriced gifts of nature (rainwater, soil fertility, fishery regeneration) that labour-time accounting systematically misses. Coherence on this axis means the scope's draws on each substance stay within that substance's **regenerative envelope** - the envelope of the ecological agent whose state the draw moves (§Shared budgets and possibility-tension), read on the element the draw is denominated in (§Material flows).

The two overlap heavily and each captures what the other leaves out. The labour view accounts for all human time, including weightless services (teaching, care); the material view accounts for nature's contributions (fishery yields, soil fertility) that carry no labour-hours until depletion forces remediation. Both axes enter the plan as **possibility-tensions**: each scope's draws press against **regenerative envelopes** (the rate at which a natural system replenishes what is drawn from it), its outflows against **absorptive capacities** (the rate at which a natural system can process what is released into it). These possibility-tensions are as hard as the labour limit - their **pull** diverges at the envelope - so no desire can buy a draw across them, however well it would score on harmony.[43]

The federation shares freely across scope boundaries; optimising coherence moves toward a shape in which, wherever possible, **the political unit lines up with a self-closing loop of production and consumption**. The planner **measures** coherence on the live flows and, where a compose/split/merge would raise it, auto-generates a proposal - which must be accepted by the affected scopes through ordinary plan-acceptance. **The flows are computed; the redrawing is decided.**

[43] Marx, *Capital* Vol. III, Ch. 48: "Freedom in this field can only consist in socialised man, the associated producers, rationally regulating their interchange with Nature, bringing it under their common control, instead of being ruled by it as by the blind forces of Nature; and achieving this with the least expenditure of energy and under conditions most favourable to, and worthy of, their human nature. But it nonetheless still remains a realm of necessity. Beyond it begins that development of human energy which is an end in itself, the true realm of freedom, which, however, can blossom forth only with this realm of necessity as its basis. The shortening of the working-day is its basic prerequisite."

The two axes draw two different shapes. Where the labour and material metabolisms close at the same boundary the scope is unambiguous; where they part, each axis pulls the cut its own way. The **labour** axis gathers activities into a **production cluster** - the human-time loop (§Two surfaces), the shape of a workshop, a clinic, a studio, indifferent to terrain. The **material** axis gathers a place into a **bioregion** - the watershed, the foodshed, the airshed, the ecoregion - the extent at which water, nutrient, and energy cycles actually close. It is **material is local; labour is universal** read on the surface: the matter-bound scope takes the shape of its watershed, the time-bound scope the shape of its collaboration, and the political border is the cut that loses on **both** - severing a production loop and a material cycle at once while answering to neither. The nature-facing branches - food, water, energy, forestry, fishing, the ecological reserve - settle toward their bioregions; the placeless ones - knowledge, design, software, governance - carry almost no material flow and federate by collaboration alone.

Custody follows the metabolism a resource lives in. A scope holds **custody** of the social property entrusted to it (§The federation and the Universal Commune) - tending it, never owning it - and because the two axes draw two shapes, custody parts along them. **Land and the ecological agents** - soil, water, forest, fishery, atmosphere - are nodes in a **material** cycle, so their **stewardship custody** settles on the **bioregional** scope: the one whose harmony carries that cycle's regenerative-envelope possibility-tension, so the steward is whoever **feels the pull** as the draw nears the limit - the people who can regulate a watershed are exactly the ones it penalises for spending it. **Means of production and skills** - tools, facilities, trained hands - are nodes in a **labour** cycle, so their **operating custody** settles on the **production cluster** that closes the human-time loop around them: the oven's custodian is the people who bake. So a single farm parts three ways - its soil and water stewarded by the bioregion, its tractors and barns operated by the producers, and its **title**, where one is still forced, held by a purpose (§The socialization of property). What private property fused into one - **ownership of land** - the federation un-fuses along the very axes that draw its scopes: stewardship to the bioregion, use to the producers, title to a purpose. And because refederation re-cuts on the live flows, custody is a **moving assignment** rather than a standing title: when a drying watershed raises the coherence of gathering a foodshed's soil and water under one steward, the proposal redraws the custody with the scope - **the flows are computed; the custody is decided**.

This puts the cost where it can be answered. Custody by the material loop sets the regenerative-envelope possibility-tension on the **steward's own** harmony field, so an over-draw is felt by exactly the scope that can ease it - there is no externality to internalise, because the cost was never external to the custodian. The market lets that cost fall downstream, on whoever has no title to charge for it; the single central plan lets it fall on no one, until the threshold is crossed all at once; here it falls, continuously, on the steward. Yet the steward **tends** rather than rules: the custodian cannot exclude from the watershed or sell it, holds voice grounded only in the labour contributed to its scope, and holds custody by a refederation that re-cuts the moment a better-closing loop appears - a moving trust rather than a territory. The alignment has one true dependency, and it is honest to name it: the steward feels the **right** pull only if the envelope is inscribed in the right place. A tipping point set too gently (§Possibility-tensions and their fragility profiles) misweighs the very limit it defends, so custody is only as sound as the fragility profile the federation has governed onto the agent - which is why inscribing those profiles is the federation's deepest stewardship responsibility, a standing charge rather than a settled matter.

The benefits compound: lower transport overhead, shorter feedback between need and provision, voice over a process whose consequences land on the same people who shape it. And at the same structural seam - **the processual abolition of the division between mental and physical labour**. When a scope's loop closes, the people who **plan** the loop and the people who **work** it are the same association. The thinking and the doing live in the same producers - by **the geometry of the closed loop**.

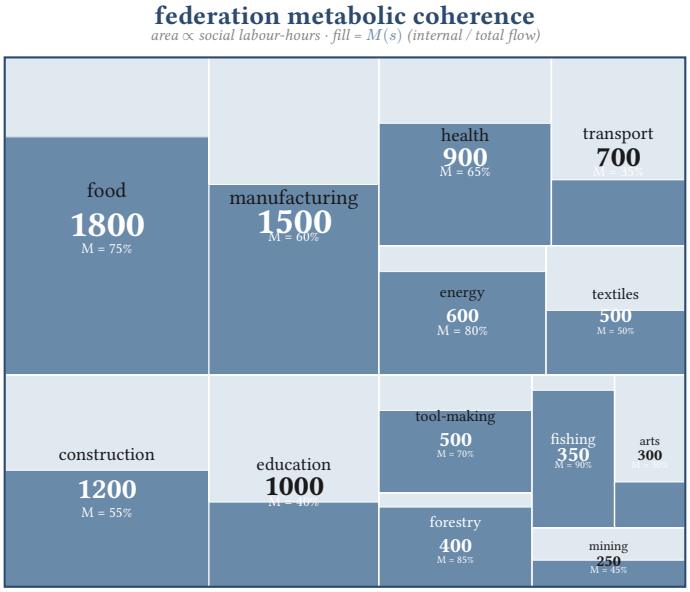

Figure 9: **Federation metabolic coherence.** Each cell's fill (steel blue from bottom) shows $M(s) = F_{\text{internal}}/F_{\text{total}}$ - the ratio of internal to total material-and-labour flows. High M (e.g. fishing 90%, forestry 85%) means the scope's production-consumption cycle is self-contained; low M (e.g. arts 30%, transport 35%) means most flows cross the scope boundary - it primarily serves or is served by other scopes. Scopes compose, split, or merge to optimise M. Plan progress (Figure 1, green) and metabolic coherence (here, blue) are independent: a scope can complete all its matched hours yet still depend heavily on external flows.

5.11. Metabolic sustainability

The federation has the strongest structural conditions for metabolic sustainability of any economic system yet proposed, across five dimensions:

Operational - the stock network is empirically more stable than markets (which oscillate as business cycles) or forecast plans (which cascade as shortage chains). Decoupling at every scope boundary, continuous variability-driven re-sizing, and metabolic coherence $M(s)$ as an engineering parameter dampen the propagation effects that destabilise other stock systems.

Material - no growth imperative, no extractive layers (profit, rent, advertising, debt service). The whole social product returns to producers and community as credit, voice, or free time. The share of social labour available for actual (re)production is structurally larger than under capitalism.

Ecological - ecological agents are first-class, with regenerative envelopes as hard **possibility-tensions** in every plan, not externalities. The atmosphere, watersheds, soils, fisheries each enter as agents with stewardship custody. No recipe that would breach an envelope can be

matched - its possibility-pull diverges first. This is the structural difference that no growth-driven economy can match.

Social - skills are (re)produced in the purple section; the catalogue is sustained as a stock. The full (re)production of **persons** - childhood, care, demographic reproduction, the affective and educational infrastructure that makes humans, not just workers - is partially captured under communal consumption but not yet first-class as reproductive labour. This is the design gap most urgent to close.

Energetic - no fossil lock-in, no profit motive to keep dirty capacity online. Renewable transitions can be planned at engineering speed, not capital speed. Atmospheric absorptive capacity, when inscribed as an ecological constraint, actively **forces** the transition.

Sustainability is not automatic. It depends on comprehensive ecological inscription (especially the atmosphere), full reproductive accounting (the personhood and care layer), energy sufficiency through transition, adequate skill stocks for long-lead-time capacities, and disciplined decoupling to prevent stock cascades. But the federation is the first architecture in which **metabolic sustainability is the system's first objective**, rather than a constraint imposed on something else - profit, growth, accumulation. Capitalism is structurally unsustainable; state socialism was partially sustainable but not by design. The federation is **sustainable by construction** - sustainability is what it is for.

6. Production and the social metabolism

The economy read as one metabolism - the moments of production and consumption, the coherence of its flows across scopes, and the matter that rides them.

6.1. Production, distribution, exchange, consumption

Bourgeois economics tears apart things that belong together - treating **production**, **distribution**, **exchange** and **consumption** as autonomous, neighbouring spheres. They are not. They are members of a totality, distinctions within a unity, and production predominates: the structure of distribution is completely determined by the structure of production.

This is the reason the social plan is *one* harmony and not four bolted-together mechanisms. The whole continuous problem is a single relaxation, and the four classical moments are simply its parts:

- **Production** - the **primal**: the recipe cascade $x = Py$ and the living labour it spends ($\text{SNLT}_{\text{direct}}$). The **feasible set** - all that can be made - and the moment that "predominates," generating the rest.
- **Distribution** - the **dual**: the pulls ($\text{SNLT}_{\text{crystallized}}$ at the binding limits), what each good *costs* and is claimed against. Twofold: of the *agents* of production (who works at what - the matching of skilled labour) and of the *products* (the credit pool, drawn by voice) - the obverse of production.
- **Exchange** - the **coupling**: the flows that realise the division of labour and carry products to where they are worked and consumed (inter-process inputs, inter-scope provisioning, transport). Internal exchange, "comprised within production" - never private.
- **Consumption** - the **objective**: the unmet desire each good relaxes, the priority-weighted want that harmony maximises ("consumption creates the need… as drive and as purpose"). *Final*

consumption injects the demand; *productive* consumption is the using-up of inputs *inside* the cascade.

Exchange continues; what changes is who holds the means it runs on. The exchange of activities and abilities that takes place within production itself belongs directly to production and essentially constitutes it, and carrying products to finish them for direct consumption is an act comprised within production itself. What presupposes private production - and what the commons leaves behind - is *private* exchange: exchange raised into an autonomous sphere, mediated by exchange-value and money. When grain flows from where it grows to where it is milled, that is exchange in the *internal* sense - the division of labour realised, the product carried to its consumption - and the scopes **support** one another through it. No one pays anyone: the social share is settled by **distribution** - the credit earned against $\mathrm{SNLT}_{\text{crystallized}}$, drawn by voice.

This is why the coordinating signal is a **pull**, never a price. A price is exchange-value: it coordinates self-interested owners across a market. A pull is marginal harmony: it coordinates one production process - the felt loudness of an unmet need, the push-back of a binding limit. And it speaks in two labours: $\mathrm{SNLT}_{\text{direct}}$ is the living labour a stage spends *this period* - the availability a pull pushes against; $\mathrm{SNLT}_{\text{crystallized}}$ is the living labour a good has gathered up its *whole cascade* - the size of the pull it exerts, and the cost against which *any* means of consumption is claimed.

Because production predominates, the structure runs one way: the recipes and their $\mathrm{SNLT}_{\text{crystallized}}$ fix the pulls; the pulls fix the allocation and the flows; the flows fix what each person may consume. Distribution is the obverse of the agents of production - here, literally the *dual* of the production constraints.

Yet "predominates" is not "precedes." Seen whole, the four are not a sequence run in turn but a single **saddle point** struck at once - production the primal, consumption the objective that summons it, distribution its dual, exchange the coupling between - and they **interpenetrate**: production is already consumption (the using-up of means and labour inside the cascade), consumption already production (the need that calls the next loaf into being). The plan does not produce, *then* distribute, *then* let people consume; it strikes all four in the one act of solving. The syllogism, read straight off the optimisation - production the **universal** (the feasible set), distribution and exchange the **particular** that mediates, consumption the **singular** in which the whole is rounded off - and the point where they balance, priority-weighted want = crystallized cost, is value itself, now as a condition of **stationarity**.

The same picture, formally — want against cost, living against dead, forward against adjoint:

$$\underbrace{L_k = \sum_i x_i\,\ell_{ik} \le A_k}_{\text{living-labour limit — } \mathrm{SNLT}_{\text{direct}} \text{ over all production, each hour once}} \qquad \underbrace{D_r \le C_r}_{\text{ecological draw}}$$

$$\underbrace{\lambda_k = \frac{t}{A_k - L_k}}_{\text{skill pull}} \qquad \underbrace{\mu_r = \frac{t}{C_r - D_r}}_{\text{draw pull}} \qquad \underbrace{x = Py}_{\text{cascade — forward}} \qquad \underbrace{\hat{\ell} = P^{\top}\ell}_{\mathrm{SNLT}_{\text{crystallized}} \text{ — adjoint}} \quad (P = (I - A)^{-1})$$

$$\mathrm{pull}_d = \overbrace{\frac{2w_d g_d}{q_d^*}}^{\text{WANT — priority-weighted desire, on means of consumption + reproduction}} - \overbrace{\sum_k \hat{\ell}_{dk}\lambda_k + \sum_r \hat{d}_{dr}\mu_r}^{\text{COST — crystallized SNLT \& draw, at the scarcity pulls}}$$

$$\text{at harmony:}\quad \underbrace{\frac{2w_d g_d}{q_d^*} = \sum_k \hat{\ell}_{dk}\lambda_k + \sum_r \hat{d}_{dr}\mu_r}_{\text{value as stationarity — priority-weighted want = crystallized cost}}$$

Read that cost term again: it carries **two** duals, not one - labour's λ_k and nature's μ_r. They are the *same kind of object* - a pull on a binding limit, in the same labour-hours - so value is **ecologically extended from the start**: a good owes the living labour congealed in it *and* the strain it laid on every **regenerative envelope** it drew from. The **metabolic rift** is then nothing exotic; it is simply a dual **climbing**. When the city eats what the country's soil grew and the nutrient does not return, μ on that soil rises and is felt by every recipe that draws there - *before* the soil is spent. And **resilience** is this very structure carried across *time*: a reserve is desire held at low priority against tomorrow's draw, sized and ranked by the same fragility profiles. What a market meets only by **crossing** it - the externality, the depletion, the stock-out - the plan carries *inside* the calculation, a second dual standing beside labour's.

6.2. Material flows

The material axis of metabolic coherence is concrete and quantitative. Every resource in the federation is denominated **twice** - once in crystallized labour, once in **matter** - and the matter side resolves all the way down to **conserved chemical elements**. This section is how that second ledger is kept: substances are first-class resources, a product's composition is read off the recipe that makes it, and the keystone invariant is that **every element is accounted for, from the input draw to its final destination**.

6.2.1. Substances are resources, and chemistry is derived

Carbon, nitrogen, CO_2, water - each is a resource specification like any other, so a substance can be contained in a product, produced and consumed by a process, and carried in the stockbook. A single **periodic table** - atomic weights, in g/mol - is the one quantity typed by hand, and everything else **derives** from it by arithmetic. A molecule is its **formula**, its atom counts, so molar masses and element mass-fractions compute straight from the table: CO_2's carbon fraction is the computed 12.011/44.009. This is the layer on which reactions **balance** by integer atom conservation.

> A LITTLE CHEMISTRY
> **mole** — a counting unit, like "a dozen" but enormous — one mole is a fixed, vast number of molecules. It is the bridge between grams you can weigh and molecules you must count.
> **atomic weight** — the mass of one mole of an element, in grams (carbon, 12.011). The whole *periodic table* is just this number for each element.
> **molar mass** — the same for a whole molecule: its atoms' weights added up (CO_2 is 44.009).
> **mass-fraction** — what share of a compound's weight one element makes up — carbon is 12.011/44.009 of every gram of CO_2.

6.2.2. Composition is read off the recipe

A crop's nutrients are exactly what the grow process drew from the soil, so a product's composition is a **consequence of its recipe**: of the substances a process consumes, the share its other outputs leave behind is embodied in the primary product, per unit of it -

$$\text{embodied}_e = \sum_{\text{in}} \text{consumed}_e - \sum_{\text{out}} \text{produced}_e, \quad \text{fraction}_e = \text{embodied}_e / q^{\text{primary}}$$

- conservation made constructive. Two ways of growing the same crop are two

recipes, hence two compositions: the **variable-composition** problem is met at the recipe level, each recipe carrying its own composition. A spec-level table serves as the **fallback** wherever a product's recipe is still to be authored, and holds the definitional identities that ground the derivation (the soil-nitrogen store **is** nitrogen).

6.2.3. Every element accounted for: conservation as keystone

The invariant the whole material ledger rests on: every tracked element a transformation consumes reappears in its outputs. Conservation here means each loss is a **named output** - runoff, flue-gas CO_2, leachate - so a recipe that sheds matter routes that matter to an explicit destination and balances exactly. Every gram thus carries an address on the books. When an element leaves a loop for a sink - nitrogen washed downstream, carbon vented to air - that drift in a nature stock **is** the metabolic rift (§Metabolic coherence), a quantity the planner reads directly.

6.2.4. The cycles, and how they couple

Conservation shows its face as a set of **cycles**: each tracked element loops through the ecosystem and closes on itself when nothing leaks. They differ in shape. Carbon and nitrogen pass through the **air** — carbon by photosynthesis and respiration, nitrogen by fixation and denitrification — so a strand can leave as gas. Phosphorus and potassium have **no gaseous phase**: they enter by the slow weathering of rock and leave mostly by water, closing through soil and the long geology beneath.

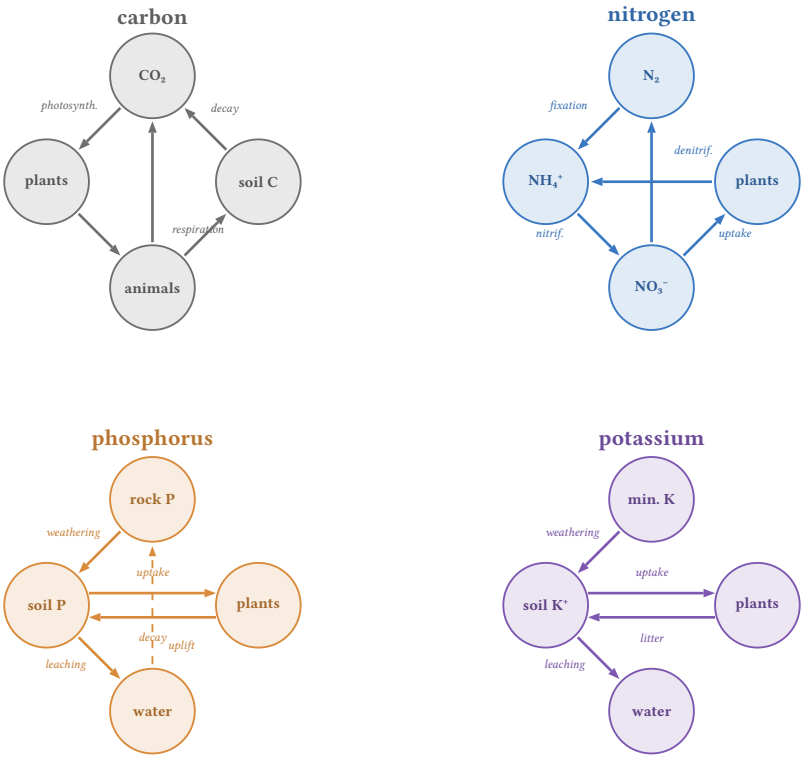

Figure 10: **The four cycles, apart.** Each tracked element loops through the ecosystem and closes when nothing leaks. Carbon and nitrogen reach the **air** — carbon by photosynthesis and respiration, nitrogen by fixation and denitrification — so a strand can leave as gas. Phosphorus and potassium have **no gaseous phase**: they enter by the slow weathering of rock and leave mostly by water, closing through soil and geology (uplift, dashed, is the long return). Same conservation, four shapes — and four different ways to leak.

Apart, they are four wheels. Together they turn one ecosystem, because **life carries them bundled**: a leaf, a body, a fall of litter is C and N and P and K at once, in the proportion its recipe fixed. The same edges carry all four colours; a transformation **splits** the bundle — decay sends carbon to the air and the nutrients to the soil — and a strand that leaves its loop is a **rift**, weighed by its own pull. Energy tells the other half: it enters once from the sun, drives the synthesis, and leaves as heat at every step. Matter **cycles** and is conserved; energy **flows through** and degrades — the closed wheel and the one-way arrow are conservation and the second law, drawn together.

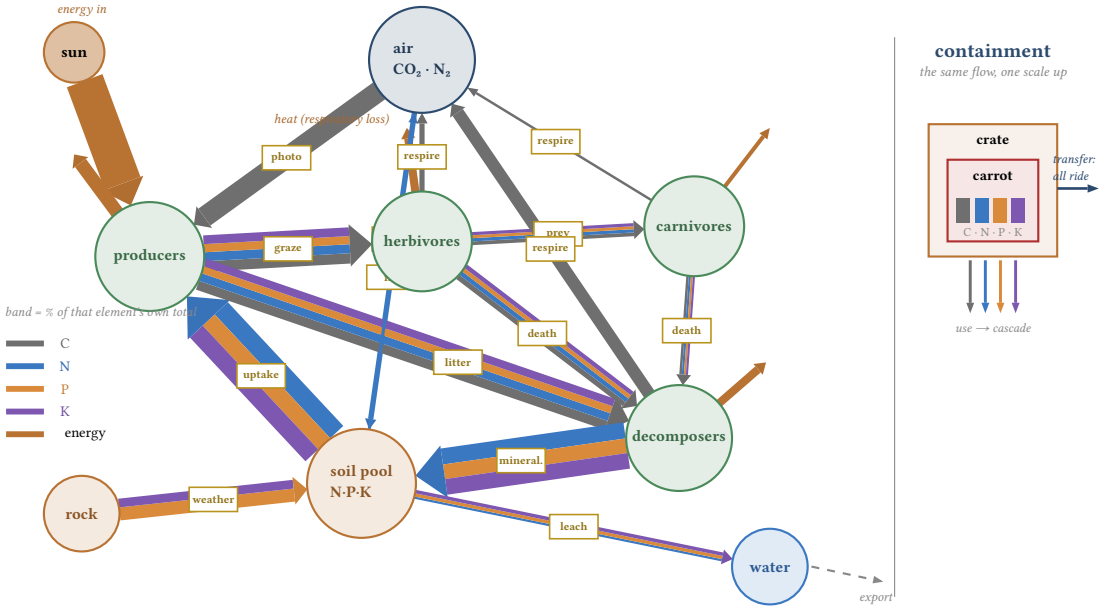

Figure 11: **One ecosystem — a graph and a proportional Sankey at once.** The node layout is the food web; **each ribbon's width is the size of the flow**, with every element scaled to **its own** total — so a band is that element's share of its own throughput, and the four cycles read at one comparable scale. Life carries them **bundled**: on each trophic edge (grazing, predation, litter, death) C, N, P, K move in near-equal shares of their own cycles — the signature of travelling together. Transformations **split** the bundle: decomposition returns nutrients to the soil, carbon to the air. Inputs enter by photosynthesis, fixation, and rock weathering; **rifts** leave by leaching to water and denitrification. Matter **cycles** and is conserved; energy enters once from the sun and leaves as heat at every step — one-way through the closed wheel. **The same pattern nests** (right): a product contains its elements (the ribbons), a crate contains products, a truck contains crates — crystallized labour and matter travel together in the contents through every box, and draw down together when the container is used (the cascade). (Magnitudes illustrative.)

Drawn bare, that same web is a reaction network — resources and the recipes that transform them, the form of the previous figure at the scale of a whole ecosystem:

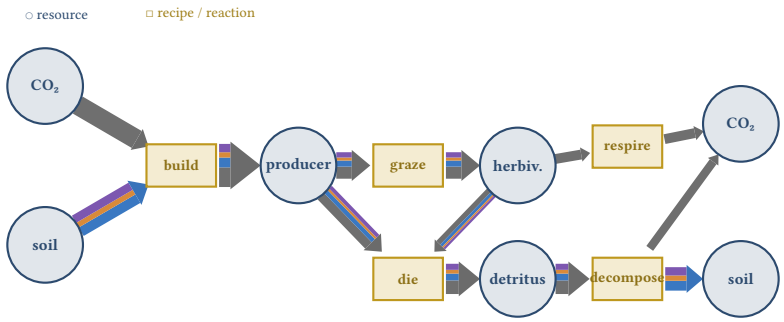

Figure 12: **The ecosystem as one reaction network.** The same metabolism in the bare metabolite–reaction grammar of the chemistry figure: resources (○) and the recipes / reactions (□) that transform them, with proportional flux split by element. **Build** (photosynthesis + uptake) draws CO_2 and soil nutrients into a producer grazing carries the bundle up the chain; respiration returns carbon to air; death and decomposition return the nutrients to soil. A plan, a balanced reaction, and an ecosystem are one object — the difference is only which rows the conservation matrix happens to carry. (Schematic widths.)

6.2.5. Balancing is the null space of conservation

Balancing a chemical equation, finding a network's independent reactions, and balancing a redox half-reaction are **the same operation**: the integer **null space** of a matrix whose rows are **conserved quantities** - one per element, plus a **charge** row whenever an ion is present - and whose columns are **species**. Build the matrix once, reduce it once, and read every answer off the null space.

> A LITTLE LINEAR ALGEBRA
> **species** — the chemist's word for each distinct substance in play — every reactant and product, one per column of the matrix.
> **stoichiometry** — the fixed whole-number proportions in which a reaction consumes and makes its species ($2\ H_2 + O_2$ makes $2\ H_2O$, never 1.7).
> **null space** — of a table of conserved totals, the combinations of species that leave **every** total unchanged. Each such combination is, exactly, a balanced reaction — so balancing is reading off the null space.
> **ion / charge** — an *ion* is an atom or molecule carrying net electric charge; that charge is simply one more total that must balance, which is why the same method handles electron-shuffling (*redox*) reactions.

One degree of freedom ⟶ a **unique** balanced reaction.

k degrees of freedom ⟶ a **basis of *k* independent reactions**: the species set spans a reaction **network**, and the basis lists every independent transformation in it. (Many independent reactions, one network - made literal.)

Charge is one more conserved row, so redox balances by the very same machinery. **Energy** (first-law, via formation enthalpies and a heat carrier) and **isotopes** (each its own element) are **documented pluggable rows**, ready to switch on the moment that data is added.

6.2.6. A reaction network is a plan

Here the two halves of the system meet. A balanced reaction **is** a recipe - reactants the consume-flows, products the produce-flows, the stoichiometric coefficients the quantities, in moles. A **network** of reactions is therefore a **plan**, and the convex engine already solves its shape: the planner's per-good balance bars are **exactly** the stoichiometric balance $S \cdot v = 0$, and a solve returns the activity levels - which **are the reaction fluxes**. Flux-balance analysis runs directly on the planner we already have. Demand a product, cap the boundary feedstocks, and the engine finds the flux distribution that meets the demand under conservation, the intermediates netting to zero (Figure, right).

> A LITTLE SYSTEMS BIOLOGY
> **flux** — the rate a reaction runs — how much it turns over per unit time. It is the chemical twin of an *activity level*: how hard a recipe is being run.
> **flux-balance analysis** — biology's standard way to find the steady rates inside a metabolic network: fix what crosses the boundary, ask for a product, and solve for the internal fluxes that leave every intermediate exactly balanced (as much made as used). It is the very same convex solve the planner already performs.

6.2.7. Labour crystallizes in the matter

Living labour combines with matter and **crystallizes** there as labour-time: value is that crystallized time and nothing besides — not a second substance riding alongside the matter, but the social measure of the hours the matter took. So the two ledgers a resource carries are matter and the labour crystallized in it, both true of the **same body**. When a resource is contained in another — flour in a sack, nutrients in a carrot, a carrot in a crate — the container holds the crystallized labour of its contents **additively**, kept separate from its own assembly labour, and holds their

matter the same way. The containment cascade emits a draw-down event for each thing inside whenever the container is drawn on, so when a box becomes something else its contents enter the new process's inputs at their own crystallized labour **and** their own element-content, carried across the boundary intact (§The two measures). The federation's coherence is the coherence of **both books — labour-time and matter — propagating as one body**.

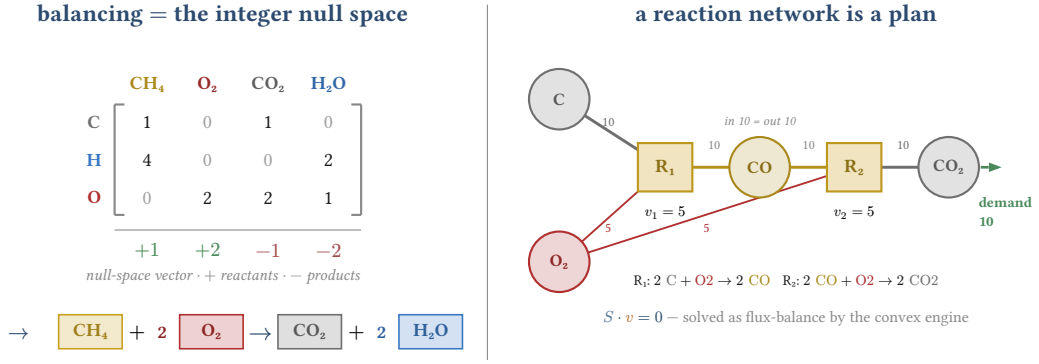

Figure 13: **Chemistry, two ways. Left:** balancing is the integer **null space** of the conserved-quantity matrix - rows are elements (a charge row joins them for redox), columns are species, entries are atom counts. The null-space vector $(+1, +2, -1, -2)$ - positive for reactants, negative for products - gives the smallest balanced equation CH4 + 2 O2 → CO2 + 2 H2O. One degree of freedom yields one reaction; k yield a **basis of k** - a network. **Right:** a network **is** a plan. Two reactions form a pathway from boundary feedstocks (C, O_2) to a demanded product (CO_2); the engine's per-good balance bars are the stoichiometric $S \cdot v = 0$, so a solve returns the fluxes v_1, v_2 directly - flux-balance analysis on the planner's own engine. The intermediate CO is produced and consumed in equal measure and nets to zero.

6.2.8. One motion: affinity and pull

Read as a balanced reaction, a recipe is a process of nature carried whole into industry. The producer sets a want and a boundary, inserts nature's own transformation between themselves and raw matter, and steps to its edge - the watchman and regulator of a process that, once understood, runs on its own terms. What stays essentially human moves elsewhere, and the equations say where.

Beneath the chemistry and the plan lies one motion. A reaction settles by descending a potential, its free energy, while conservation pins every atom. The plan settles the same way - descending its harmony objective $\mathcal{H}$ (priority-weighted desire-deficit and labour) while the balance $Sv = 0$, capacities, and regenerative envelopes hold. Two systems, one descent under conservation:

$$\text{nature:} \quad \min_{v} G \quad \text{subject to atoms, charge, energy conserved}$$

$$\text{plan:} \quad \min_{v} \mathcal{H} \quad \text{subject to } Sv = 0, \text{ capacities, envelopes}$$

Hold a conserved quantity and read its pull: the **marginal change in the optimum per unit** of it - the worth of one more. This one object wears two names. Physics calls the pull of free energy a species' **chemical potential** μ_i; the plan calls the pull of harmony a resource's pull μ_r - one construction, named for whichever potential it is the gradient of. That is why what it drives is the same, even though a chemical potential is counted in joules and a pull in labour-hours. A reaction's **affinity** is the drop its potentials leave across the arrow, $-\sum_i \nu_i \mu_i$; an activity's net pull is the

gap its pulls leave across the recipe. Each system advances an activity while that gap stays open, and rests where it closes:

$$\underbrace{\sum_i \nu_i \mu_i = 0}_{\text{a reaction at rest}} \quad \Leftrightarrow \quad \underbrace{\frac{2 w_d g_d}{q_d^*} = \sum_k \hat{\ell}_{dk} \lambda_k + \sum_r \hat{d}_{dr} \mu_r}_{\text{an activity at harmony}}$$

The left is chemical equilibrium; the right is value as a stationarity condition. A good's pull is the plan's chemical potential, an activity's net pull its affinity, and the plan's optimum an equilibrium whose potential carries human ends.

One caution keeps the picture honest. A living economy, like a living cell, never sits at that dead equilibrium where every affinity has gone to zero - throughput holds it off, a **sustained steady state** fed at its boundary, not a reaction run to exhaustion. What only ever rises is the dissipation it sheds, $\sigma = \sum_j \mathcal{A}_j v_j \geq 0$; the "rest" the solve reaches is the **optimum of the program**, the point where no activity can buy more harmony, not the stillness of a spent reaction. Equilibrium is the shape of the resting condition; the metabolism lives a little above it, and the gap is exactly the work it is doing.

The phrase *a potential that carries human ends* holds the whole distinction. The constraints are the **is** - the conservation nature hands us, indifferent to any wish. The potential is where the **ought** enters: free energy is given, while harmony is authored, with desire and priority written into its shape. Matter descends to the floor of whatever well it occupies; the plan selects the well, by choosing what to want. So as the producer steps aside from the mechanism - which now relaxes itself, the way matter does - the human takes the one seat physics leaves open: **authorship of the potential**, the setting of desire, priority, and the fragility profiles that weigh each limit. The descent runs on its own; choosing its destination is the realm of freedom.

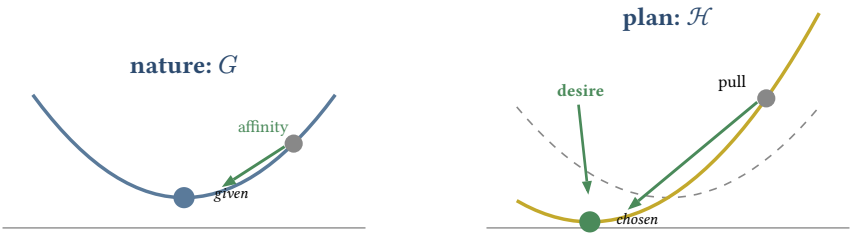

Figure 14: **One descent.** Nature and the plan both reach rest by sliding to the floor of a potential, the slope itself the force - a reaction's affinity, an activity's pull. The walls are conservation, shared by both: the **is**, with no vote taken. The floor is where they part. Nature is **given** its well and settles wherever it lies; the plan **authors** its well - desire reshapes the landscape (dashed ⟶ solid) so the resting point falls where human ends place it. The floor's location is the **ought**.

The metabolic rift reads cleanly in the same light - the second law crossing a boundary. Every transformation sheds disorder; a closed loop takes it back, an open one ships it to a sink. The pull μ_r on a regenerative envelope is the weight of that shedding, felt before the sink overflows - the thermodynamic bill carried inside the calculation, settled at the source.

One object, then, wears two faces. The reaction we **observe** is the **is** - what the metabolism did, read from the event ledger. The reaction we **plan** is the **ought** - the flux that should run to meet a desire under conservation. Between them stands the pull: nature's measure of how far a process

sits from rest, and the federation's measure of how much a want is worth - the same number, read forward as policy and backward as fact.

To master a process this way is to author it. The conserved quantities bind absolutely - the producer obeys them to the letter - and will enters at the one open place, the potential. Mastery is obedience to the law and authorship of the end at once, each deepening the other: a natural process is commanded best by one who understands it well enough to set it toward a chosen well and let it relax there. Freedom here is the **composing of necessity** - nature, fully grasped, made to run toward human ends on its own terms.

This rests the whole weight of the result on the act of authorship. Matter settles wherever its well lies, beyond praise or blame; an authored potential answers for its own shape. So the federation owns its outcomes whole: every limit either carries a pull or stands open, and an open one is a choice someone made. The metabolic rift, read here, is a potential that weighed a limit too lightly. Freedom and responsibility become one coin - which is why inscribing the fragility profiles, weighing each edge before it is reached, is the federation's deepest and most continuous charge.

And as production becomes the running of natural processes, understanding nature and organising human life converge into one act: the grasp of a reaction and the allocation of it are the same object, held by the social body that knows it. The wealth of such a society is measured by how much it understands, and how much free time that understanding returns, as living labour settles toward watching and regulating. The more fully the **is** is mastered - the conservation, the reactions, the descent that now runs itself - the more plainly the **ought** stands out as the work of will alone, the one supply reserved past every law and every computation. How matter behaves settles every question but one - what to want - and desire alone answers that. So mastery of nature, carried far enough, magnifies the human: the more of the world runs itself, the more it falls to us to say where it should run. It clears away the realm of necessity - the labour borne because it must be - and leaves standing the realm of freedom, the choosing of ends for their own sake. Humanity steps to the side of the engine and into the one seat that was always, irreducibly, its own.

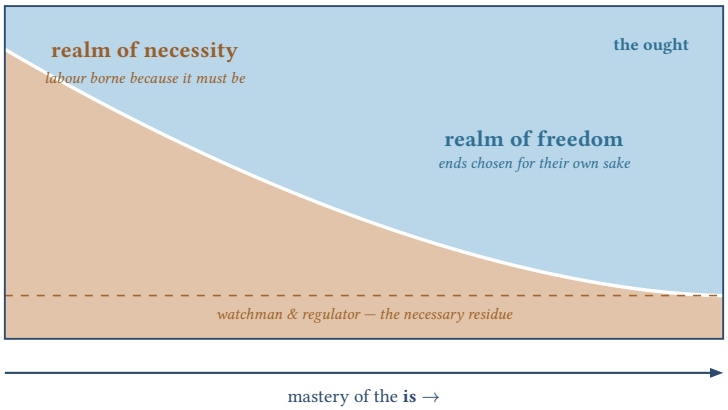

Figure 15: **The horizon.** As mastery of the **is** deepens - more of the metabolism understood, the descent running itself - the realm of necessity contracts toward a floor (the watchman and regulator's residue) while the realm of freedom opens above it. The whole is one society's time and energy; the boundary marks how much of it necessity still claims. The work that stays essentially human rises into the room it clears: the authorship of ends, the **ought** that no law or computation supplies.

7. One program: space, time, and the computability of the plan

Everything above describes a single harmony to relax. This section makes the structural claim explicit and draws its consequence: the whole federation - every place, every period, every scope, every recipe - is **one convex program**, and that program is **computable on ordinary hardware**, because it is convex and sparse. The century-old objection that a planned economy cannot be calculated is a mistake about the mathematics, and the mistake can be named precisely. What the solve produces is the plan; the moment-to-moment runs as the **metabolised** cascade beneath it (§Derived demand, §Reserves). **Solved** and **metabolised** divide the work between them - one sets the targets and the pulls, the other realises and adapts them between solves.

So how much computer does it take to hear the planet? The engine's measured throughput sets the bill. One ordinary machine solves a refined regional plan - on the order of **180,000** located activities - to **99.8%** of demand met in about **100 seconds** (the native compiled kernel; the OSQP backend is several times faster again). The plan decomposes by scope - the contained-in tree, a two-level scheme - into independent subproblems of that size, coordinated by a master over a handful of rounds. The planet's hardware bill is then arithmetic: size a node to a region of a few million people, and the globe is a few thousand of them.

scope	people	servers	one refinement
a region (measured)	≈ 3 million	**1**	≈ 100 s · 99.8% met
a nation	≈ 70 million	**≈ 25**	a few minutes
a continent	≈ 1 billion	**≈ 350**	a few minutes
the globe (district-refined)	≈ 8 billion	**≈ 2,600**	< 1 hour · a few rounds

Figure 16: **The hardware bill for a planet.** Sizing a node to the measured regional solve and scaling by population, the entire world's plan - refined to the **district** - runs on a **few thousand** commodity servers, fewer than a single ad-tech company runs to place its banners. The decomposition is embarrassingly parallel, so wall-clock is set by the coordination **rounds** — and those are set by the **diameter of the cover**, not by the number of scopes or the size of the problem, which is why the cluster buys nothing a badly-cut federation cannot squander; and because a single combined solve is itself tractable (§Inter-scope propagation), the cluster buys **speed and refinement**, never mere feasibility. The century-old objection that a planned economy cannot be calculated was never mathematical. It was political.

7.1. Space, time, and scope are coordinates

There is no separate "spatial planner," "scheduler," or "federation engine." A good is a resource specification **indexed by where and when**: grain in the country at harvest is a different located-dated good from the same grain in the city next month. **Where**, **when**, and **in which scope** are coordinates on the same field; the plan is one allocation across all of them at once. The recipes themselves are **placeless and timeless** (a RecipeFlow carries no location and no date), so **where and when to instantiate each one is a free variable the optimum chooses** - site selection and timing fall out of the solve rather than being decided in advance.

7.2. Everything is an activity

Production, transport, and storage are **columns of one technology matrix**. A recipe run at a place-and-period is an **activity**: it consumes some located-dated goods and produces others, for a labour cost. **Transport is just such an activity** - it consumes bread in the country and produces bread in the city, the haulage its living labour. **Storage is the same activity pointed through time** - it consumes bread this period and produces it next, the holding its labour. So storage is transport through time, with one difference that is not a detail but a law: **time has an arrow**. You may ship a good in either spatial direction, but you can only carry stock **forward** - there is no activity that moves a good from the future to the present. Space is symmetric; time is not.

This gives the plan its shape: a **spacetime lattice** whose spatial axis is a **tree** (the contained-in hierarchy of places and scopes) and whose temporal axis is a **chain** (period after period, linked only forward by storage). One optimisation traverses the whole lattice.

A period is a length of time, not only a place in the order. The chain says which period follows which; it does not say how long one lasts, and the difference is not bookkeeping. A means is **used** rather than consumed, so what the plan rations is its **time** (§Means of production) - and time is a quantity a period must therefore **have**. Two ovens across a year are not two ovens across a day, though the count is the same in both; what differs is the hours the plan may occupy and hand back. So a means' capacity is its **stock times the time it stands available**: three ovens available eight hours are twenty-four oven-hours, and a bake that occupies one for ninety minutes takes an hour and a half of them. Read without the time factor, capacity collapses to a headcount and the plan rations the means' **substance** - the one thing §Means of production says is never what is rationed.

How long a period is, is a governed choice. A federation planning in days and one planning in seasons are answering different questions with the same machinery, and neither length is more correct: the granularity is a political-economic decision about how finely a scope wants to see - and to be held to - its own commitments, made by that scope and ratified like any other. What the plan requires is only that the length be **stated**, because every capacity on the lattice is denominated in it.[44]

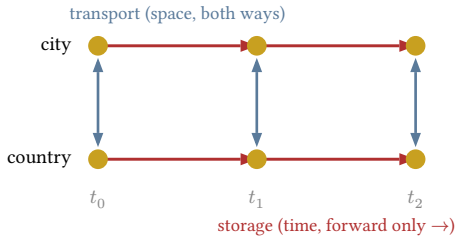

Figure 17: **The spacetime lattice - and everything on it is a column.** Each node is a good at a place-and-period, made by production there. Transport carries it across space within a period (either direction); storage carries it forward through time (one direction only - time's arrow). Production, transport and storage are **all the same kind of object**: columns of one technology matrix. Space is a **tree** (the contained-in hierarchy); time is a **chain**; the plan is one solve over the whole grid.

[44]This is why a worker's availability and a machine's are one declaration and not two: an availability window over a period yields hours, and hours over a stock yields capacity. §"Worker, soil, and tool: one field" reads them as one form at different scales, and they are only literally one form when both are time-denominated - a body's hours and a tool's hours, drawn against the same period. A stock counted without its time is the point at which the twin breaks.

7.3. Transport and storage are one recipe - the container decides space or time

Push the last sentence further. If transport and storage are **the same activity over different arcs**, they ought to be **the same recipe** - and they are. A move, of cargo or of a person, has one shape: `pickup` a good, `dropoff` it, `use` a **container** (drawing its service), `work` a **driver** (drawing labour). It compiles through the ordinary recipe compiler and tiles through the ordinary arc machinery - there is no transport subsystem, only relocation recipes the plan composes from.

Which arc - space or time - is not declared; it is derived from the container. A container classified keep-in-place (a warehouse, a silo) relocates its contents through **time**; a mobile one (a vehicle) - or none at all (walking) - through **space**. You never label a recipe "storage": a warehouse-recipe is simply useful only on a time arc, a truck-recipe only on a space arc, and the solve finds which. And "stationary" names what *using* the thing does. Relocating the warehouse itself is another recipe, in which the warehouse is the cargo and a heavy-hauler the container. **What it takes to move something lives in its classification, not in a flag**: heavy goods select a costlier recipe; everything is movable, it merely has one applicable recipe and that recipe is dear.

Loss is a consume, sized by the manner. A good in transit or store loses some of itself - spoilage, breakage, evaporation. This is not a number stamped on the good; it is **the same shape as a tool's wear**: a `consume` on the relocation recipe of the good's own lost fraction, exactly as wear is a consume of a durable's condition. Refrigerated storage loses less than a warm shed - **same milk, different recipe**. The manner carries the loss.

$$\underbrace{g_a \to g_b}_{\text{relocate — one column}} \qquad \underbrace{(-\delta g, -c)}_{\text{loss \& wear — further consumes}} \qquad \Rightarrow \quad \hat{\ell}_{g_b} = (1+\delta)\,\hat{\ell}_{g_a} + \ell_{\text{haul}}$$

So storage, transport and their losses add **no new machinery** to the program: they are columns and consumes in the **same** technology matrix T, and the adjoint $\hat{\ell} = P^{\top}\ell$ crystallizes their labour into the good exactly as for any other input - the loss δ and the worn condition c ride up the cascade like a bill of materials. **Storage is transport through time**: the only difference is whether $g_a \to g_b$ crosses space (a vehicle) or a period (a warehouse).

A container is known by a code, and its label is the code printed. A resource has a field for a tracking identifier, and the design fills it at declaration with a code that is short, unique within its scope, and silent about owner and contents. The label on a bin, a crate or a van is that identifier as a scannable code, so the thing on the shelf and its line in the ledger are joined by one read. A place contains places (site, room, shelf) and a container contains resources a container that moves takes along what was where it was, and contents that state no place of their own are read where their container is. What a label states beyond the code follows the good's specification: a food states its designation, its ingredients and its date, and the date says which it is, best-before or use-by. A count of the container is the same read followed by a figure, and a count that finds the ledger off lands the difference from the ledger of its instant, and every count keeps the figure it found. A photograph can propose the rows of a shelf, each confirmed or struck by the person; the picture stays with the one who took it, and its hash rides the count, so that a member who doubts a count may contest it, the counter attaches the picture, and the franchise decides, a void correcting every part of the count; the design is `docs/observation/capture.md`.

None of this is *faux frais*. That category belongs to the capitalist **form** - the overhead of circulation, the money-commodity, commercial bookkeeping - and even there transport itself counts as **productive**. A transparent plan has no circulation sphere: carrying bread to the city and keeping it

to next week are **real social costs of a distributed society**, living and embodied labour counted like any recipe's. The social ledger names where committed time goes - direct work, transport, training - and calls **none** of it false. The only true partition is committed time against free time.

7.4. The worker is a rider

A person travels on the very same lattice - with two differences. Their located stock is a **presence**; and where that presence rests they **supply labour**, gated by it - her supply $\mathrm{Lab}(s, c) \leq h \cdot p_c$, with p_c her presence (itself relocated by the move above, carried forward as storage) and h her hours. So a doctor far from the clinic is the same assignment puzzle: the plan either carries the patients to her, **carries her to them**, or trains a second doctor - one optimisation weighing transport against formation, with the car a scarce shared resource and the driver a particular person whose hours are wanted elsewhere too. She rides the same recipe that moves cargo, save that she can also **propel herself**: walking is a relocation recipe with **no separate driver**, its cost her own transit time. A travel **mode** is such a recipe, held in the scope's store like any other and instantiated per rider; its `work` flow resolves from its own `pickup` - the mover is her own motor, the skill she bears at the mode's pace - so walking spends her hours alone and a driven ride adds the driver's `work` beside hers. Every scope holds the three baseline modes - walking, a bicycle and a car - and a shed adds its own; each carries the router's costing for its manner and a process specification of its own, so a ride performed names its mode. And she is **one** person: her rides are whole-count acts decided on the discrete layer's tree, she leaves a cell only from the presence the band found there, and she is always somewhere - so the plan places her, band by band, rather than spreading her. A **means** travels the same way: a durable classified `mobile` is a mover whose **service** follows its presence (supply $\leq$ cap $\cdot\, p_c$, the twin of the labour gate), so a shared tractor or a projector stands in one place per band, and the plan says which. A band is as fine as the scope declares it (`period-spans`, the band dial - a day of three bands is three periods), and a mover's hours are each band's own: her roster entry for that band, the machine's availability window met against it. So a two-hop day is a morning field, a midday ride to the middle one, an afternoon at the third - each ride whole, each departure from where the band found the machine, and a two-hour band carrying two hours less the ride.

Travel modes - walk, cycle, ride - are gated by **skills**. One who cannot walk lacks the skill and must be carried; **accessibility is the generic rule** - a crate, lacking "walk" no less than a person who cannot, takes a vehicle by exactly the same logic. Skill decides which recipes apply; need and cost decide among them.

And the rider's hour falls into one of **two ledgers that must not be confused.** When the trip is **planned work** - getting a worker to where their labour is needed - the transit is real social labour, a genuine cost of the spatial configuration, and the driver's work is productive. When it is **consumption** - a ride taken for one's own ends - it is a **means of consumption**: a `deliver-service` good that drivers produce and a person draws **in free time, against credits** (the ride's crystallized SNLT - the claim rule of §The credit identity); the rider's own time-budget is never touched. Society plans that *enough rides exist*, not that a given person rides at a given hour.

Automation removes the **living** labour from the move. A mode with no driver - and no skill to require - carries even one who can neither walk nor drive, its only cost the **vehicle itself** (embodied labour). What automation removes is the **living** part of the cost: it turns a driver's hours into a machine's embodied hours and **returns the freed time to the people** - which is the whole point of the plan.

7.5. The measure of the arc

Every spatial arc carries the territory's own measure. The federation hosts its own router[45] over the maps the commons already keeps, and reads from it, per pair of located places and per **manner** of moving - walking, cycling, driving, the shared bus - two numbers: the trip's **time** and the road's **distance**. Each manner is a production function for the same relocation: walking spends labour-power alone; a bicycle adds a tool's wear; the bus spends a driver's work and a **communal** container's service, its timetable the availability window of a shared means. The two numbers then scale the recipe's flows **by their physics**: `work` and `use` are spent over the trip's time, fuel and loss over the road's distance - a congested ten kilometres that takes as long as fifteen costs fifteen kilometres' worth of driver-hours and burns ten of fuel.

And the measure learns. Every performed journey - one process based on the mode it was made by, its `pickup` and `dropoff` recorded as ordinary events - is a **measurement** of its arc, under the manner its mode's recipe carries; recent trips weigh heaviest, and where measurements exist they speak ahead of the router's estimate. So the lattice's transport coefficients converge on **lived** time: the federation learns the socially necessary time of its own relocations from its own events, with rush-hour and weekend arcs keeping separate entries where timetables make departure matter. Plan and observation then diverge **per arc** exactly as they diverge per process - one ledger, seen from space as from time. A ride the plan asked for, once performed, is recorded the same way: the trip is observed, and the ride's own rows (the vehicle's service in use, the rider's hours) are confirmed like any other; where the rider is comes from the position the person states or shares. The router remains knowledge of the territory; the table of measured arcs is what the federation has **learned** of it.

7.6. It is activity analysis - and it is already ValueFlows

The mathematics of this is **activity analysis** (Koopmans)[46]: a rectangular **technology matrix** T whose columns are activities and whose rows are located-dated goods, with T_{ga} the net amount of good g that activity a produces (negative if it consumes). The plan is the vector x of **activity levels**.

This structure **is the data itself**. The ValueFlows knowledge layer already *is* activity analysis: a RecipeProcess is a column, a RecipeFlow is one matrix entry, a ResourceSpecification is a row, and the standard VF **action** of each flow (`produce`, `consume`, `work`, `move`) fixes that entry's sign and role. The planner is a function of the recipe graph: it **compiles** the flows into T, tiles the columns

[45] Valhalla - the same self-hosted, multimodal engine that serves the map's journeys - built over the commons' shared map (OpenStreetMap) and the transit timetables (GTFS) a city already publishes. One matrix call prices every pair of located places; the solve reads coefficients only, and an empty table falls back to plain geometry - the program is identical, the numbers merely truer.

[46] Koopmans built this formalism: the wartime routing of Allied shipping (the transportation problem - spatial activity analysis before the name), then the 1951 *Activity Analysis of Production and Allocation*, and the 1975 Nobel shared with Kantorovich. His central theorem is a duality - an allocation is efficient exactly when a price vector supports it - and he read it as the case for coordinating **through prices**: announce them, let activity managers respond. The planning procedures that followed (Arrow–Hurwicz, Malinvaud) adjust prices step by step - first-order methods, the market's own algorithm formalised. Kantorovich carried the other reading: solve the plan itself, keep the duals as transparent valuations. The federation keeps Koopmans' skeleton and completes Kantorovich's reading - an explicit convex objective (harmony) over the same constraints, the primal solved directly by a second-order method, the pulls a by-product read for transparency and for stitching scope solves together, and decomposition chosen for the polity along the very seams the math parallelises.

across the lattice, and solves.[47] Where two columns produce the same good, the federation has a **choice of recipe**, and the optimum picks the mix - the rectangular matrix makes "which recipe, where, when" a single continuous decision. ValueFlows specifies every stage from Recipe to Plan to observed event but leaves **how** a Recipe becomes a Plan open; this is that step.

$$\underbrace{y_g = \text{init}_g + \sum_a T_{ga} x_a}_{\text{delivered (located-dated balance)}} \qquad \underbrace{L_c = \sum_a \ell_{ca} x_a}_{\text{labour at cell } c} \qquad \underbrace{D_{rc} = \sum_a d_{rca} x_a}_{\text{ecological draw}}$$

delivered, labour and draw are all LINEAR in the activity levels - this is the fact everything else rests on

7.7. Two surfaces, one matrix: the row-dual and the column-dual

The technology matrix T has **two** axes, and the federation reads the plan along **each**. Its **rows** are resource specifications - **what is made**; its **columns** are **activities** - RecipeProcesses run at a place-and-period, **the making itself**. The whole L is one number; **how you sum it** is a choice of axis, and the two axes are the two things the federation has always needed to see.

Sum down the columns and you get the **scope surface** (Figure 1): activities cluster into scopes - food production, manufacturing, health - and the cells are **branches of doing**. A scope *is* a cluster of activities: scope = a contiguous block of columns. This surface carries producer voice (labour is spent on activities, so the voice it earns lives on the columns) and metabolic coherence (an $F_{\text{internal}}/F_{\text{total}}$ of the column-cluster). Production has no *a-priori* taxonomy - its tree is the **clustering dendrogram of the activity graph**, grouped by shared goods, which is exactly what $M(s)$ measures and refederation re-cuts.

Sum across the rows and you get the **classification surface** (Figure 2): resource-specifications nest into a classification taxonomy - food → grain → bread - and the cells are **kinds of thing made**. This surface carries consumer voice and desire (wants nest, so the priority weights run *down* this tree), split by the four categories of reproduction and the two distribution rails. Its tree *is* a-priori, because wants nest where activities do not.

So **scope = column-aggregation, taxonomy = row-aggregation, of the one matrix.** The two surfaces are the **two faces of the single activity-analysis**: a resource-spec (one row) is made by several activities (several columns), and an activity (one column) touches several specs (several rows). This many-to-many is exactly why **no single treemap can nest both** - a treemap nests one axis, and T is a matrix. Two figures, one object; and each carries the voice that belongs to its axis - producer on the columns, consumer on the rows - so the "how it is made / what is made" duality of governance (§Governance) is, underneath, the **column/row duality of T** itself.

The matrix is a grammar: subject - verb - object. Each RecipeFlow - each filled cell of T - is one **sentence** of the plan. Its **subject** is the **activity** that acts (the column); its **verb** is the VF action (`produce`, `use`, `consume`, `work`) that fixes the cell's sign and role; its **object** is the resource specification acted upon (the row). "Food **produces** bread"; "health **works** medical-skill"; "energy **uses** a turbine." The whole plan is a **corpus** of such sentences, and the two surfaces are just the two ways to **parse** it: collect the sentences by their **subject** and the corpus sorts into activities

[47] In the engine the pipeline carries three names, worth keeping straight when reading the code: a RecipeProcess with its RecipeFlows compiles into a **template** - inputs, outputs, and labour folded into one column-pattern, the thing actually tiled; tiling that template across the cells, one per place-and-period, yields the **activities** - the realised columns of T and the entries of the level vector x. So: recipe-process → template (compiled) → activity (located).

(the column-dual - branches of doing); collect them by their **object** and it sorts into resource specifications (the row-dual - kinds of thing made). Subjects carry producer voice (you earn say where you **act**); objects carry consumer voice and desire (wants are wants **for** objects). The verb is the metabolism between them - the living labour ℓ a sentence spends. Grammar and matrix are the same structure: a subject-by-object table of verbs is **exactly** a column-by-row matrix of actions, which is **exactly** T.

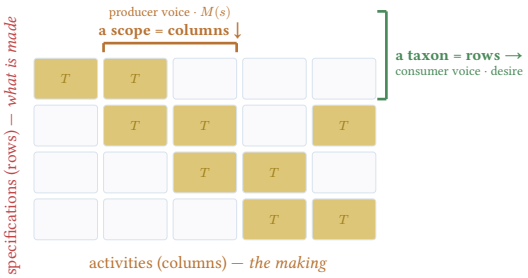

Figure 18: **One matrix, two surfaces.** The technology matrix T: rows are resource specifications (*what is made*), columns are **activities** (*the making*), each filled cell a RecipeFlow. Summing **down a block of columns** gives a scope - a cluster of activities, carrying producer voice and metabolic coherence; summing **across a band of rows** gives a specification band, carrying consumer voice and desire. A spec is made by many activities and an activity touches many specs, so the two stay distinct - they are the row-dual and column-dual of the **same** object.

The object is a noun; its classifications are adjectives. One subtlety completes the picture, and it is the difference between a **partition** and an **overlay**. The matrix's two axes are both **partitions**: every flow is performed by exactly one activity (its column) and, by ValueFlows, **conformsTo** exactly **one** resource specification (its row) - the lowest useful **kind** of the thing.[48] So each fold lands every labour-hour in exactly one cell, and **both folds sum to the same total** L - that equality is the proof the surface never double-counts.

A specification's **classifications** are a different thing: VF gives each resource a **list** of them - `food`, `perishable`, `local`, `staple` all at once - drawn from taxonomies, facets, or plain tags whose structure VF deliberately leaves out of scope. They are **adjectives** on the noun, and they **overlap**: a loaf that is both `essential` and `perishable` belongs under **both**. This is what governs how they may be counted. **You can sum nouns; you cannot sum by adjective** - add the `essential` hours to the `perishable` hours and the loaf is counted twice, because it is in both. So the classification view is a **browsing overlay**, not a third partition: invaluable for "show me everything `perishable`" or "everything `care`," but its bucket-totals deliberately **exceed** L and must never be added as if disjoint. Rendering classifications as a **tree** (one parent per node) is itself a choice - a single spanning path picked out of what is really a many-parented DAG - convenient to draw, accounting-safe only once you either fix one primary path per resource (recovering a partition) or refuse to cross-sum the facets. The surface that follows keeps the distinction visible: the activity and specification axes **partition** (their tiles tile L exactly); the facet axis **overlaps** (its tiles, summed, exceed L, and it says so).

[48] ValueFlows, *Resources*: "An economic resource or a flow can have only one **resource specification**... the lowest level useful type or kind of the resource," but "can have **any number** of **resource classifications**... used to filter, match, or group." The specification is the noun (one); the classifications are adjectives (many). Our engine mirrors this exactly - `conforms-to` is a required single reference, `classified-as` is a list.

7.8. Labour is a good on the lattice - and the matching falls out of the solve

The labour row above ($L_c = \sum_a \ell_{ca}\, x_a$) is the aggregate view: total hours at a **cell** — here a **spacetime cell** (place, period), a point of the lattice, not a rectangle of the surface — held under a cap. Refine it one step and the **assignment of people to work stops being a separate problem**. An agent's time is **itself a located-dated good**: the hours a worker offers of a given skill at a cell - call it $\mathrm{Lab}(s, c)$ - exist only where and when they are available, the same perishable, time-arrow structure as any other located good. So labour is just more of the matrix:

- a **worker supplies** it - a column producing $\mathrm{Lab}(s, c)$, capped by the agent's hours (read off their availability window);
- a **process consumes** it - its `work` flow draws ℓ hours of $\mathrm{Lab}(s, c)$ for the skill it names;
- and the per-cell cap becomes a **balance**, one per (skill, cell):

$$\underbrace{\sum_{w} x_w}_{\text{workers supply}} - \underbrace{\sum_{a} \ell_{a,s} x_a}_{\text{processes consume}} = s_{s,c} \geq 0$$

This balance **is the mediation between labour and the plan.** Pushing a process up relaxes a good's desire-tension but tightens **this** possibility-tension $-t \log s_{s,c}$; the plan settles where the two pulls meet. Its dual - the **located labour pull** $\lambda(s, c) = t / s_{s,c}$ - is the value of an hour of skill s at cell c: scarce skill, tiny slack, large pull. Weighted against each good's crystallized hours of that skill, $\sum_k \hat{\ell}_{gk} \lambda(k, c)$, it states every good's **pull** in **skilled, located** labour-time - value as a balance of tensions, now resolved per skill and per cell - while the crystallized hours themselves stay at coefficient one.

Two terms finish it. Each agent has **one time budget** spread across whatever skills they hold - a multi-competent worker is simply one who is primaryAccountable for several skill EconomicResources[49] - so their supply columns share a **single hours cap**. And a small convex **EMP congestion-tension** $\rho(\sum_s x_{w,s})^2$ - rising marginal disutility - makes work spread **evenly among all who can do it**.[50] With these in the one program, **who works which slot is read off the solution**: the worker-column levels x_w are the assignment. The two-layer matching of §Matching is, for the cooperative plan, not a separate stage - it **is** these located labour goods and the EMP tension, relaxed together with everything else.

[49] VF (actions.md): a `work` flow carries no resource, only the provider agent; "the type of work or skill involved can be identified by a resource specification." A skill is thus a ResourceSpecification; an agent holds one as an EconomicResource (conformsTo the skill spec, primaryAccountable the agent); the work flow's effortQuantity is the hours. The planner reads exactly these — skill from the recipe's `work` flow, hours from the agent's availability window — and emits the plan back in VF's two stages: a work **intent** per process-need (provider unbound — what is *intended*) and a work **commitment** per worker that `satisfies` it (provider bound — who is *assigned*).

[50] Two readings of "evenly" are a value choice the federation can set: the congestion on a worker's **total** hours equalises everyone's *total burden* (equal matched-hours-per-citizen — a multi-skilled worker eases off any one task because their other work already counts); a congestion **per skill** instead gives a uniform *share of each task* independent of one's other loads. And "evenly" means evenly *absent a declared preference*: a worker may register a skill-time preference (eager — / reluctant +), a small cost that tilts who does what without overriding need. It is read equal and bounded: clamped to a governed range $[-\beta, \beta]$ and centred on the worker's own mean over the skills they hold, in the congestion term's own unit $2\rho/H$, so it reorders that worker's own assignment and moves nobody's total load (*The voice field*).

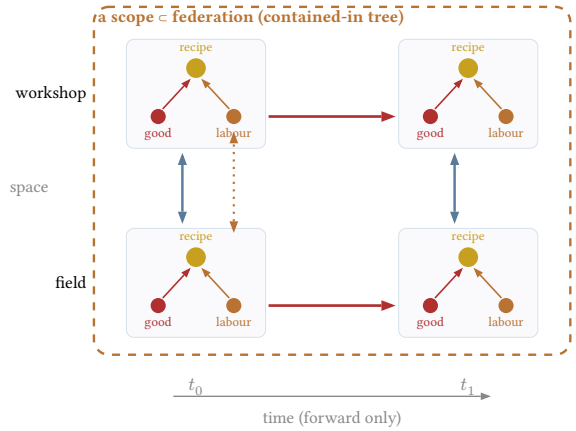

Figure 19: **Anatomy of the labour–spacetime–scope lattice.** Every located thing lives at a **cell** = (place, period) — a **good** (resource@cell) and now **labour** (skill@cell) **alike**; cells nest in **scopes**, the contained-in tree that **is** the federation. **Within** a cell, located labour and inputs feed a recipe. **Between** cells, the same `move` activity in three guises: goods **store** forward in time (one way — time's arrow) and **transport** across space (either way); labour **commutes** across space (either way) but — the strictest perishable — **never stores forward** (an unspent hour at t_0 is gone). One convex program is indexed by **all four coordinates at once**; the plan is the activity levels on this lattice, and its pulls are the located prices — of goods **and of skilled labour** — that couple it up the scope tree.

7.9. Concrete and abstract labour: the located hour and the universal substance

Putting labour on the lattice puts **concrete** labour there: a worker's hour of a named skill at a cell - **living** labour, $\mathrm{SNLT}_{\mathrm{direct}}$, spent here and now, and the **strictest** inhabitant of the lattice (it can commute across space but, alone among goods, cannot store forward at all - an unspent hour at t_0 is simply gone). But there is a **second** labour, and it is **not a point on the lattice**: **abstract** or **universal** labour-time, the substance and the measure of value. You cannot point to a cell and find it - the way you can find grain or a baker's hour - because it is not located at all.

It is what you get by **integrating the concrete labour off the lattice.** Carry the located living labour ℓ **up the technology adjoint** $T^{\top}$ through the whole production cascade, and the labour **congealed** in a good - its crystallized $\mathrm{SNLT}_{\mathrm{crystallized}}$ - is a single number with no place and no time:

$$\underbrace{\hat{\ell} = T^{\top}\ell}_{\text{crystallized labour-content — every hour at coefficient one}} \longleftarrow \underbrace{\ell(s,c)}_{\text{the located living hour — concrete, at a cell}} \longrightarrow \underbrace{\hat{\ell}_g}_{\text{the universal labour in good } g \text{ — placeless, timeless}}$$

$$\underbrace{\mathrm{pull}_g = \sum_k \hat{\ell}_{gk}\,\lambda(k,c)}_{\text{the located pull — } \lambda \text{ weights the content, never enters it}}$$

The located hour $\ell(s,c)$ is **concrete** - a quantity **at** a cell, a spacetime thing. Summed up the adjoint it becomes the **universal** labour-content of a good: the invariant that **survives the projection off the lattice**, every skill's hour entering at coefficient one. The located pull $\lambda(s,c)$ is concrete too - a **pull** at a cell - and it **weights** that content rather than entering it: a good's pull is its crystallized hours of each skill, each weighted by that skill's scarcity at that cell. So universal labour is the **numéraire** - the one dimension the whole field is denominated in (every tension and every pull,

in labour-hours). It is independent of the lattice exactly the way a **unit** is independent of what it measures: it is the axis you measure **along**, never a coordinate **within**.

And the same reading values **every** limit, not labour alone. Each good's balance and each ecological envelope carries a dual of its own - its possibility-pull $\mu(g)$, the marginal harmony that one more unit would resolve. The harmony's stationarity balances desire against **both** families of pull at once,

$$\underbrace{2wg/q^*}_{\text{marginal desire}} = \underbrace{\sum_k \ell \cdot \lambda}_{\text{located labour pulls}} + \underbrace{\sum_r d \cdot \mu}_{\text{material \& ecological rents}}$$

and **both sides are read off the one solve** - value **is** the dual of the plan, never a second computation. The crystallized $\hat{\ell}$ is the **labour-content** of that pull: what $\mu(g)$ reduces to when nothing but labour is scarce. When a natural limit binds, a good's pull rises above its labour-content by an **ecological rent** - nature's scarcity speaking in labour-hours, not anyone's work.

Follow the dimensions of that stationarity and the numéraire names itself. The left side is denominated in social desire

- weight times gap - so every dual on the right is a rate of **desire**:

$\mu(g)$ is the voice-weighted want that one more unit of g would resolve, and the value of anything is the desire elsewhere that it displaces. Labour-hours are the customary scale because labour is the wall that desire presses hardest and longest; the watershed's rent enters the same measure the moment the watershed genuinely binds, at a weight the federation ratified. Value here is a social relation made a quantity by the solve: **foregone possibility, as the federation has chosen to feel it, denominated in each other's need** - every wall entering through a ratified weight, every want through a voice. And the measure polices itself: no rewrite of a citizen's asks obtains a bundle worth more of it

- a lie moves a bundle along the isovalue surface, never up it - so what

misreporting can never command is precisely other people's weighted need.[51] That gap is information, not distortion: a good is dear because it leans on a strained limit, exactly the signal a plan that must **raise the floor, never cross nature's ceiling** needs to surface.

This is the classical distinction between concrete and abstract labour, made spatial-temporal. **Concrete** labour is the living act - this skill, this place, this hour - bound to its cell; **abstract** labour is the social substance, **placeless and timeless**, common to all commodities - "but crystals of this social substance."[52] The lattice does not **add** a labour dimension to the field; it reveals that labour is at once the **most** located thing - a perishable good pinned to a cell - and the **least** located thing - the universal measure standing off the lattice - and that the adjoint $T^\top$ is the **bridge**: it carries the living, located hour into the universal substance value is made of. The human lives on both sides too: a subject with **one** life-time budget, reckoned over the whole federation as social free time f

[51] `SolverCertificate/MultiSupply.lean multi_supply_value_dominant`; the burn-meter's `moved` and the ⚖ editor's declaration-time read are the same duals, read per solve.

[52] Marx, *Capital* Vol. I, Ch. 1, the "crystals" passage cited under *Reproduction labour and the two measures of labour*: the labour that makes a coat and the labour that makes linen are different **concrete** useful labours; as values they are **abstract** human labour, "human labour-power expended without regard to the form of its expenditure." Concrete labour creates use-value; abstract labour, measured in time, is the substance of value. The lattice realises both: concrete labour is a located, perishable good at a cell; abstract labour is what the technology adjoint distils out of it - the SNLT congealed in the product, with no place and no time.

$= 1 - L/(NT)$ - never a cell, and the universal labour that value is made of is **that same human time, summed.**

This off-lattice substance has a name: it is the substance of the **social individual.**[53] Universal abstract labour and social free time are its **two faces.** Abstract labour-time $\hat{\ell}$ is the social individual **as producer** - human time-power expended in common, the substance every commodity is "but a crystal" of; free time f is the social individual **as such** - the time set free, in which "the free development of individualities" unfolds. They are complements of **one** total human time, $L + f \cdot NT = NT$, and **both stand off the lattice** - reckoned over the whole federation, never at a cell - because the social individual is not located: it is the **sum** of all human time, split into the part the plan must still spend (necessary, concrete, located labour) and the part it has **released**.

That recasts the whole lattice as a **means.** Concrete located labour is the **cost**; the social individual in the time set free is what it is **for.** So the free-time thrift in the harmony - the whole drive to meet desire with the **least** living labour - is not an accountant's economy: it is the planner **shrinking what the lattice must claim** so as to **widen the free time in which the social individual develops.** The lattice spends concrete labour at cells; what it does **not** claim falls back to the social individual as its own time. The end of the program is to make the lattice claim **less.**

7.10. The whole plan in one solve

For a century the planning debate assumed the economy is **too coupled to solve directly** - that you must break it into pieces and coordinate them through some signal, whether the market's prices (Hayek), a planning board's faked prices (Lange), or a dual-decomposition's iterated multipliers. We found this is **false, and we can say exactly why it was believed**. Coupling means that improving the plan requires several activities to move **together** - grow less here, more there, ship the difference - and no single one of those moves improves the plan on its own. A **first-order** method (follow the gradient one coordinate at a time) therefore stalls, and the stall **looks** like a law of the problem. It is not. It is a property of the weak method. A **second-order** method reads the curvature - the Hessian's off-diagonal entries are precisely how the coupled activities move together - and takes the joint step in one shot. The monolithic problem, solved with the right method, simply **solves**. Decomposition becomes a **choice** - taken for **scale** (the monolith is near-linear but bounded by one machine's **memory**, and the split **by scope**, **run in parallel across machines**, is the efficient engine) or for scope autonomy - never a **necessity for tractability**; and the pulls are a **by-product** of the single solve - the same pulls the federation uses to stitch its scope solves together. The intractability was never in the economy. It was in believing a weak algorithm was the only one.

[53] Marx, *Grundrisse* (Notebook VII, "Fragment on Machines"), the passage quoted in full under *Governance*: once labour in the direct form ceases to be the well-spring of wealth, the measure becomes the reduction of necessary labour to a minimum and the development of individuals in the time set free. The two off-lattice quantities of this section - universal labour and free time - are the producing and the developing faces of that same social individual.

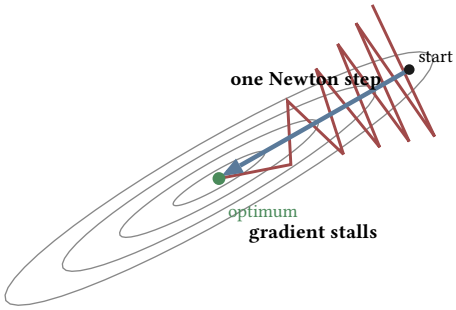

Figure 20: **Why coupling does not** force **decomposition — the proof, geometrically.** A **coupled** plan is an ill-conditioned valley: improving harmony needs several activities to move **together** (along the valley floor), but every single-coordinate step points **across** it. A **first-order** method (red) - follow the gradient, or adjust one pull at a time - zig-zags and crawls; this is the stall that for a century looked like a law of the economy and sent everyone reaching for prices to coordinate the parts. A **second-order** step (blue) reads the **curvature** - the Hessian's off-diagonals *are* the coupling - and reaches the optimum in one move (exactly so for a quadratic model; on the federation's barrier objective, in the short sequence of such steps - the ∼ 5-19 below - that the primal-dual climb takes). Same problem, same data; the only difference is the solver.

7.11. The program, exactly

Because y, L and D are linear in x, the whole plan is **bounded desire** over **log-barriers of linear forms** - a convex program with an **analytic** gradient and Hessian. Desire enters through its two members (*Honest by shape*): the fractional-gap quadratic on the scope's shared dimensions, and the satiating reciprocal - the **cap** - on each citizen's own wants, whose ask is a wall among the barriers rather than a number in the value. A small free-time term rewards spending **less** total labour, so the plan makes only what someone wants:

$$h = \underbrace{-\kappa \sum_{c} L_c}_{\text{free-time thrift, prunes waste}} \quad \text{with} \quad h = \underbrace{-\sum_{d} w_d {g_d}^2}_{\text{shared dimensions: unmet desire}} \underbrace{-\sum_{p} w_p/(1+s_p/c_p)}_{\text{own wants: bounded pull, the ask a wall}} - \underbrace{\sum_{\text{limits } \ell} \rho_\ell(-\log s_\ell(x))}_{\text{possibility-tensions; } \rho_\ell = \text{swept } t \text{ (hard caps: labour, balance, } \geq 0) \text{ or fixed } w \text{ (fragility: ecology, capacity, care ... — felt a}}$$

$$H = \underbrace{\sum_{d} B''_d \, a_d a_d^{\top}}_{\text{desire curvature}} + \underbrace{\sum_{\text{limits } \ell} B''_\ell \, a_\ell a_\ell^{\top}}_{\text{limit curvature}}$$

over the activities x, every tension — desire and limit — contributes one **rank-1 outer product** of its sparse row a

A **primal-dual** interior-point method closes on the true optimum — and because the program is convex there is only **one** to close on, so what remains between the iterate and it is a **margin**, never a wrong basin. It carries the pulls as first-class unknowns **beside** the plan: each step pairs a **predictor** toward the optimum - the affine direction, on this objective simply $-\nabla h$ - with a **corrector** that recentres, both reusing the one Hessian. It carries its own **certificate**: the **duality gap** μ - the pulls against their slacks, averaged over the m constraints - is driven down to a **floor**, the final central-path parameter, so the plan is provably within a known margin of the best possible, and the **stopping rule is that margin**, not a path-parameter to tune. Not to **zero**: the barrier weight is walked down to $t_{\min} = 0.005$ and no further, so $m \cdot \mu$ is an **irreducible** term and the certificate says so rather than promising a limit the solver does not take. Measured across a 16

-scope federation, $m \cdot \mu \approx 3 \cdot 10^{-2}$ per scope — small, and never nothing.[54][55][56] The pulls are the constraint multipliers; at the optimum priority-weighted want = crystallized cost, which is value as a condition of stationarity. The genuinely **discrete** choices - a minimum batch that must run at scale or not at all, an indivisible facility - are a thin outer wrapper: **branch and bound** over the on/off configurations, each node one convex solve of a relaxation. A node prunes on its own certified bound - the solved value less the gap the certificate proves, which is what makes the pruning sound rather than merely plausible - so an exhausted tree is a **certified global** optimum, and a budget cut-off reports the gap it actually has. The convex certificate is not adjacent to the discrete layer; it is what licenses it. A recipe states its own discreteness in ValueFlows terms — a `minimumBatchQuantity` on the process, read against the recipe's outputs into a floor in runs, and a `whole-count` classification for an act that happens a whole number of times — and a plan made from records opens the wrapper on either, reporting what the search decided beside the certificate it rests on. Placement is the same decision: a person read as a mover, or a means classified `mobile`, carries a unit presence whose rides are whole-count acts, so where each stands in each band is decided on that tree — one move per band, from where the band found it, and always somewhere — and the plan reports the placement beside its commitments. Two facts about the search are the scope's to state and to read. Its **node budget** is a governed number, set for one solve beside τ and the hold (`node-budget`), and the certificate carries the budget beside the nodes it spent — an exhausted tree is certified, a spent budget says how many of its nodes and how far the gap stands, and a budget spent before any whole plan is found says so rather than dressing the relaxation as a decision. And a **federated** solve keeps the relaxation, and says so: the ADMM's guarantee —

[54]The margin is **not** μ alone, and saying so was this engine's own error for a long while. Weak duality gives the bound only through a pair: for any feasible x, any $y \geq 0$, and $r = \nabla f(x) - A^\top y$, $f(x) - f(x^*) \leq y^\top s(x) + \|r\| \cdot \|x - x^*\|$ — convexity plus Cauchy–Schwarz, two lines. When stationarity is **exact** the second term vanishes and the margin is $y^\top s(x) = m\mu$, which is the textbook statement. When it is not — the ordinary case — μ **bounds nothing on its own**, and a certificate quoting it alone is quoting half an inequality. Measured on a located program: $m\mu = 0.11$ against $\|r\| \cdot D = 1.41$, so the residual term was thirteen times the one being reported. The engine now computes the whole bound, with D a radius derived from the constraint rows themselves, and reports $+\infty$ honestly where the feasible set is unbounded and no finite bound follows. Verified against finite differences and brute-force optima in `tests/planner/test-kkt-verification.scm`.

[55]**A stalled point is not a plan.** Certification decides "done" on one ladder for slacks and for stationarity alike: 10^{-6} relative stationarity is the noise floor (`optimal?`, reported), $\approx 10^{-3}$ is where the path-follower's own endpoint sits and is a plan, and above 10^{-1} the solve came apart and the point goes to the path-follower when one is in hand. Measured live on Berlin's day: the primal–dual returned after 23 iterations at stationarity 1.0, feasible, every maximin wall slack by hundreds, $\tau = 0.08$ under futures capturing 0.35 - and passed on feasibility alone. The path-follower's rescue ladder answers a failed Newton direction the same way: a "native-c (jacobi)" rung after the first attempt whenever the automatic preconditioner was a factor - IC(0) stalled the three-day program at nine steps, stationarity 0.89, certified bound $4.8 \cdot 10^8$; Jacobi converges it in 272 steps at 10^{-4}, bound 11. A solve that ends stalled names the tightest walls it sits on, so the log says which. And the preconditioner is judged by what it does: the residual the inner CG leaves over $\|g\|$ - two dot products already in hand - says whether the preconditioned solve solved the Newton system at all, and a factor that leaves more than half of it behind is retired for the rest of the solve, the step redone matrix-free. No shape is named and no threshold on the pattern; the KKT residual is the whole test.

[56]This is exact for the **primal–dual** method described here — it halts on $\mu = s^\top y/m < 10^{-6}$ and reaches $\approx 10^{-8}$ in practice, a real gap on a real certificate. The engine also carries a **primal path-following** backend, used where the primal–dual step does not fit, and there the guarantee is a different shape: it has no μ at all, walking a barrier weight t down to a floor $t_{\min}$ and stopping. That floor is not a rounding detail — the barrier still standing at it is **felt in the plan**. Against a limit that is **not** binding, its residual $+t/x$ gradient is opposed only by the free-time thrift $\kappa\ell$, so an activity settles near $2t/(\kappa\ell)$ **above** what was asked of it. Spread across an economy that is a first-order quantity: on a 240-sector input–output fixture at $t_{\min} = 0.02$, 87% of activities ran inflated, for 2.15 × the total activity and 1.6 × the labour of the same delivered demand. So on that backend the floor is a **governed** parameter traded against solve time — 0.005 costs 19% more time for 26% less labour at identical delivery, and is now the default. Companion: *The Shape of the Plan* §8, Obligations 3b and 5b.

separate solves settling at one pull, converging to the joint optimum — is a convex one, a boundary pull read off a scope's tree is not that scope's dual, so a commune whose recipes declare a batch or a whole count is solved across the boundary to the digit and marked `relaxed`, never mistaken for decided. Deciding across a boundary (each tree against the standing pull, the pull settled again, a duality gap to report) is a research thread, and the marker is where it would land.

The program can be asked backwards. Every scarcity in it is a constant on a balance row - opening stock, a raw's draw bound, a cell's hours - and a constant becomes a column by construction: an **acquire** column, $+1$ on the row and priced in hours, so the solve buys exactly what the walls it holds require and nothing else. Which walls it holds is one table: a **tier** names, for each kind of tension, whether it stands soft as stated, as a wall at its ask, or not at all - rights alone (the floors), rights and needs (held margins at their ease), all three (every want at its ask) - and one row more holds **nothing**, because a barrier solve sits a little inside every wall and buys a little even with nothing to hold; every bill is read net of that residue. Three solves give three **bills**, and the step between them is the price of a tier in hours and in kind: forty hours of cooking a week, two hundred kilos of beans, a cold room. The ecological walls take no acquire column unless named - what nature bounds is what the bill cannot buy - so a tier whose certificate reads infeasible has hit such a wall, and says which. A bill is the cheapest mix at the margin: a floor on what the tier needs and the resource to look at first, never a recipe, and under uncertainty each row is read at the future that asks the most of it. It turns a plan for what a scope has into a claim on what it lacks, addressed outward - to the federation, to the neighbouring commune, to a vote about hours - and a household reads the same three numbers off its purse.[57]

7.12. Sparse by nature, and therefore computable

The Hessian above is a **sum of rank-1 terms over the constraint rows**, and those rows are **exactly the recipe graph**: a recipe touches a handful of goods, not millions. So the problem is **extremely sparse** - the federation never imposes this sparsity, the recipes hand it over. This is the whole reason it scales. The Newton step never forms H; it only ever needs the product Hv, and

$$Hv = \sum_{\text{limits } \ell} B''_\ell \, a_\ell \left(a_\ell{}^\top v \right) \qquad \text{costs } O(\text{nonzeros}) \text{ - linear in the size of the recipe graph}$$

so a preconditioned conjugate-gradient solve rides the sparsity directly. Putting the pieces together gives the **complexity of planning the whole economy**:

- The program is **convex** → a single global optimum, found in **polynomial time** - no combinatorial explosion (the discrete wrapper is a thin shell on the genuinely-indivisible few percent).
- Interior-point methods converge in a number of Newton steps that grows only as $O(\sqrt{m})$ in the worst case[58], and in practice is **small and slowly-growing**. On a **favourable** structure

[57] Realized in `module/playnet/planner/solve/engine.scm`: `*acquire*` and `acquire-columns` (laid before the prune; a bought unit is supply on the row, never a producer), `%tier-shapes` read by `tier-shape` at every site that emits a tension, `bills-fold` as the one residue-then-net loop under `plan-bills` (records), `scope-bills` (a live scope) and `family-bills` (the purse), which the family face draws as "what the week would take". `docs/planning/socialization/three-bills.md`; `tests/planner/test-bills.scm`: the diet fixture starved of labour with hours for hire reads 9.7, 10.2 and 16.5 hours for the three tiers, a land tip below what feeding three people costs leaves even the rights tier without a plan, and the Berlin chain with no surplus set out bills 64.7 kg of surplus for its wants alone.

[58] The $O(\sqrt{m})$ bound is the **self-concordant barrier** complexity of Nesterov and Nemirovski (*Interior-Point Polynomial Algorithms in Convex Programming*, SIAM, 1994): path-following damped Newton on a log-barrier with parameter $\nu = O(m)$ converges in $O(\sqrt{\nu})$ outer steps. Naming it places this engine precisely - the **path-**

- a production chain, each stage feeding the next - we measure it **flat at $\sim$ 19 steps** across a 250 $\times$ size range; on a realistically **spatial** economy - located production with transport and storage, a space-time mesh - a block-sparse mesh still **centres in $\sim$ 20-26 steps**[59] even a **degenerate** one at a **flat optimum** - which stalls a plain path-follower at its iteration cap - the **primal-dual predictor-corrector** clears in **the same $\sim$ 5-19 steps**, the gap μ certifying the halt with no centring left to grind.[60] The inner solve stays cheap either way - gradient-regularized and **matrix-free**, a **handful** of conjugate-gradient iterations - so **the solve stays near-linear and meets demand in full**. The realistically **sector-rich** economy bears this out: a 240-sector input-output table over 40 provinces and 8 periods - **176,000 activities**, dense commodity hubs and all - solves in **five** primal-dual steps where a direct factorisation is defeated outright. Tractable throughout.
- The cost lives in the **inner** linear solve, and its difficulty depends on **how the economy is coupled**. A near-**banded** chain a structure-aware preconditioner holds near-constant. A **space-time mesh** couples activities across the whole lattice and **ill-conditions** that solve - but the cure is **matrix-free**: a **gradient-regularized** Newton step (its regularizer scaled to the gradient norm, so it fades to zero at the optimum - after Doikov–Nesterov) holds the conditioning down, measured to **cut inner iterations by orders of magnitude** (7,214 $\rightarrow$ 9 conjugate-gradient iterations) and the solve by **4.9 $\times$** (6.5 $\rightarrow$ 1.3 s) on a located mesh of $\sim$ 18,000 activities, while **raising** demand met from 98.4% to 100%. What a single global **direct factorisation** cannot do is **fit**: its fill grows faster than the data (we measured one balloon to **minutes** where the matrix-free iteration took **seconds**), so past some resolution the model exceeds one machine's memory. **That** is the honest lever for **decomposition**: split the economy **by scope**, solve each scope's small problem, and coordinate the scopes only through pulls on the goods they **provision across their boundaries** - a located labour-pull on each good that passes between scopes. Those scope solves are **independent** - so they run **in parallel**, across cores and across machines. And decomposition is not only a parallelism device but a **complexity** one - a **two-level scheme**, outer pull-coordination over inner per-scope solves, that sidesteps the monolith's pessimistic worst case rather than merely spreading it across hardware: the standard lesson that worst-case second-order bounds

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following branch of globalized Newton, a **primal-dual** damped Newton on a self-concordant barrier, the problem class held apart from the cubic- and contracting-regularization branch. The gradient regularization above is the one tool borrowed from that other branch, and only to condition the **inner** solve, never to globalize. The **predictor-corrector** that clears a degenerate flat optimum is Mehrotra's (1992) - the standard **primal-dual** interior-point method - which sets the centring from the **measured** gap μ rather than a fixed schedule, so the flat optimum that stalls pure path-following converges in the same handful of steps.

[59] The count is a function of how far the central path is walked, so it is quoted at the **shipped** floor. Measured across 704–53,400 activities: 17–20 steps at $t_{\min} = 0.02$, 21–26 at the shipped 0.005. The tighter floor is worth those steps — it is what stops a non-binding limit leaving an inflated activity behind (§7.11′s footnote) — and it does not slow the **located** engine, where the same A/B moves solve time by less than the run-to-run noise (70,976 activities: 7.31s at 0.02, 7.40s at 0.005).

[60] True of the degeneracy that method is **aimed** at — a flat optimum where centring from the **measured** gap beats a fixed schedule: measured at 272 path-following Newton steps against **six** primal–dual iterations on one program. It is not true of every flatness. On a near-free-transport program — transport so cheap that the optimum is a plateau in the transport directions — the primal–dual returns an **infeasible** point: a barrier breached at -4.2×10^{-2} slack, objective twelve orders worse, where the path-follower returns a good feasible plan it merely cannot **certify**. Two different degeneracies, two different right answers. The engine guards the difference — no route may return an infeasible plan, and a breach falls back to the path-follower, which cannot cross a barrier by construction. Note also that nothing **routes** on flatness: the dispatch selects on hub density (whether the preconditioner wants a matrix-free step), so a flat program reaches the primal–dual only by coincidence.

overstate the cost of **structured** problems, and that the structure is best exploited by a coarse outer loop.

Writing n for activities, m for constraints, z for the technology matrix's **nonzeros** (the recipe-graph edges - and $z = O(n)$ for a sparse economy, since each recipe touches a handful of goods), and κ for the **coupling** (the conditioning of the Newton system), the whole cost decomposes by stage:

stage	cost	set by
assemble — compile, build, index	$\Theta(z)$	one linear pass over recipe flows — measured flat, ≈ 0.01 ms/activity
Newton step — matrix-free CG	$\Theta(\sqrt{\kappa}\, z)$	Hv costs $\Theta(z)$; CG needs $\approx \sqrt{\kappa}$ - a **gradient-regularized** step holds this to **tens** (measured 6–40 × fewer)
Newton step — direct factor (OSQP)	$\Theta(\text{fill})$	fill $= z$ banded $\to n^{1.5}$ mesh $\to n^2$ dense; sidesteps κ
outer steps N — primal-dual	$\tilde{O}(\sqrt{m})$, ≈ 5-19 in practice	predictor-corrector; the gap μ certifies the stop — clears even a **flat** optimum in the same few steps
monolith — banded chain	$\Theta(Nz) \approx \Theta(z)$	$\kappa = O(1) \to$ near-linear (the favourable case)
monolith — space-time mesh	near-linear, $\approx n^{1.0-1.1}$	gradient-reg holds inner CG to a **handful** per step; Newton bounded ($\sim$ 5-19, **flat optima included**) — z stays dominant (measured to 66,000 activities)
monolith — sector-rich I-O	near-linear	dense commodity hubs defeat a direct factor; **matrix-free primal-dual** in **5 steps** — 176,000 activities (240 sectors, 40 provinces, 8 periods) in $\sim$ 2 min
scope decomposition — S scopes, P cores	$O(R\, S\, \text{solve}(z/S)/P)$	$R \approx 30$ rounds on a low-diameter mesh; a long chain is $O(\text{diam})$, **Anderson-accelerated** $\sim 3\times$
memory	$\Theta(z)$	$\approx$ 88 bytes per nonzero (direct-CSR)

Figure 21: **The complexity of planning, by stage.** The whole solve is $\Theta\big(N \cdot \sqrt{\kappa} \cdot z\big)$ - **structure** z (the matrix nonzeros, $O(n)$ for a sparse economy) times the **interior-point loop** N ($\approx$ constant) times the **coupling** $\sqrt{\kappa}$. A production chain has $\kappa = O(1)$ and is near-linear; a space-time mesh raises the **inner** conditioning κ - which a **gradient-regularized** Newton step holds down **matrix-free**, no dense factor - while the **outer** count, even at a **flat, degenerate** optimum, a **primal-dual predictor-corrector** holds to a **handful** (the gap μ certifies the stop), so the mesh too is **measured near-linear to 66,000 activities** - and a **sector-rich** economy of 176,000 activities, dense commodity hubs and all, solves in **five** primal-dual steps where a direct factor is defeated outright. It is then **memory**, not arithmetic, that forces **scope decomposition**, linear in scopes. The discrete (minimum-batch / indivisible-facility) wrapper is **branch-and-bound** over the 2^B on/off choices, each node one convex solve pruned on its own **certified** bound. Where the relaxation lands **integral** no branching is needed at all ($B = 10$, 1,024 configurations, **one node**). Where every column lands **fractional** - the case branching must earn its keep on - nodes grow **near-linearly** in B against an exponential configuration count: $B = 12$ visits 19 of 4,096 configurations, 99.5% pruned, **certified global**, in six seconds. The node budget is therefore never the binding limit; **wall time** is, since each node is a full convex solve and the program grows with the horizon. What ends certified discrete planning is the per-node solve, not the search.

This is the crucial turn, and it **unifies the computation with the politics**: the spatial economy is solved as the **federation itself** - scopes planning locally, coupled only through pulls on what they share - and that decomposition is **exactly** what makes it parallel. Not a hope but a **measurement**, this engine: a realistically-coupled **national** economy - $\sim$ 156,000 activities, 240 sectors over 40 scopes across 8 periods, an input-output table's resolution tiled over a country's provinces - solves on one laptop core - the gradient-regularized engine is **near-linear** on a **located** spacetime mesh (measured $\sim$ 143,000 activities in 70 s, demand met in full); a **sector-rich** input-output coupling is heavier per activity - its basic-commodity hubs are dense - which is exactly what the **decomposition** below is for - at the engine's own optimality certificate; and its scope solves run **concurrently** - on a 6-core laptop the parallel decomposition runs **3.7 $\times$ faster** than serial, at the same plan ($\geq$ 99.8% of demand met), the speedup widening with more cores and - because each scope's data and solve are self-contained - **across separate machines**. The same federation that keeps the **authority** distributed (next section) is the very structure that keeps the **computation** distributed. **Memory is never the wall** - the national plan fits in $\sim$ 240 MB; what scales is **time**, and time divides across the federation's machines. The calculation objection is answered twice over: the optimal plan of an economy is a **large sparse convex program** - and it is **decomposable along the very seams of self-governance**.[61]

[61]Cockshott and Cottrell argued the feasibility of in-natura planning on 1990s hardware using sparse iterative methods on input-output systems; the federation's harmony is the convex-optimisation generalisation - desire, ecology, space, time, and recipe choice in one program - and inherits the same near-linear scaling, now with an optimality certificate. Our engine confirms it empirically. A **production chain** is the favourable case: a matrix-free interior-point solver with an incomplete-Cholesky preconditioner stays near-linear to 180,000 activities and beats an industrial ADMM solver (OSQP). A **spatially-coupled** economy is the harder case — a plain matrix-free step pays $\sim$ 7-10 $\times$ per activity there — and the **gradient-regularized** step holds the located transport/storage mesh **near-linear too** (measured $\approx$ 140–320 μs/activity, **flat at nine Newton steps** out to 143,000 activities), so the monolith runs without a dense factor. And a **sector-rich** input-output economy - dense commodity hubs that defeat a direct factor outright - is solved by a matrix-free **primal-dual** step (Mehrotra's predictor-corrector) in a **handful** of iterations: 176,000 activities (a 240-sector table over 40 provinces and 8 periods) in **well under a minute**, the outer count **flat at six** and the duality gap certifying the stop. Scale beyond one machine is then a **memory** limit, met by **scope decomposition**: each scope solved in full by a compiled interior-point kernel, the scopes run in parallel (a coarse-grained shared-memory solve, measured 1.9 $\times$ on 2 cores, 4 $\times$ on 6, the per-scope solve bit-identical to the serial one), coordinated by a pull-coordination (ADMM) whose round count is set by the **diameter of the cover**, not by the number of scopes — boundary information moves one hop per round, so a cut that makes a good travel k scopes to reach its eater cannot settle in fewer than $\sim k$ rounds, and that is information propagation rather than a weakness of the coordinator. Measured on one economy cut two ways: as a **chain** (diameter $S-1$) the count climbs $23 \to 35 \to 47 \to 105$ for $2, 4, 8, 16$ scopes — meeting the demand in full throughout — and only at 32 scopes does the coupling outrun the round budget, hitting the cap at 86% met (given the rounds it asks for, 247, it reaches 99.8%); re-cut as a **star** (diameter 2, same demands, same land, same grain crossing boundaries) it is **flat at about thirty-three rounds and 99.9% met from two scopes to thirty-two**. Degradation is therefore a **cliff** at the point diameter exceeds the budget, not a slope. So the federation scales without limit **provided the cover stays low-diameter**, and the cover is a **governance** choice with a computational price attached. Refederation on metabolic coherence (§4.1.3) pushes hard in that direction, by the mechanism that matters: a long cut forces a good to cross **every** intermediate boundary, so the same physical demand generates far more boundary traffic and every middle scope becomes mostly a conduit — which is precisely what a low $F_{\text{internal}}/F_{\text{total}}$ measures. On the economy above, the chain cut moves 9,806 units across boundaries where the star cut moves 1,283: coherence sees the difference and prefers the star. It is a strong **proxy**, not an identity — coherence is a volume ratio and diameter is a path length, and a ring and a star can carry equal crossing volume at diameters $S/2$ and 2. Maximising coherence attacks the dominant cause of a long cover; it does not bound the diameter. **Anderson acceleration** buys roughly 3 $\times$ on top (27 $\to$ 9 rounds at two scopes, 103 $\to$ 29 at four) and **rescues** convergence outright where a long chain would stall: at eight scopes the plain iteration hits its cap at 89.6% met where Anderson closes in 31 rounds at 99.7%. Two things the round count does not say. First, what a federated solve can **prove**: a certified bound over **every** coupling-feasible plan, measured at 1.5% of the objective on a 16-scope star and 15% on a 16-scope chain — meaningful on a well-cut cover, merely honest on a badly-cut one, and in both cases a **margin**, not a proof of exact optimality. Second, that the coordination converges the **prices**, not the quantities: at the stop the scopes still disagree about what crosses, so some scope plans on goods its neighbour is not sending. The plan is near-optimal

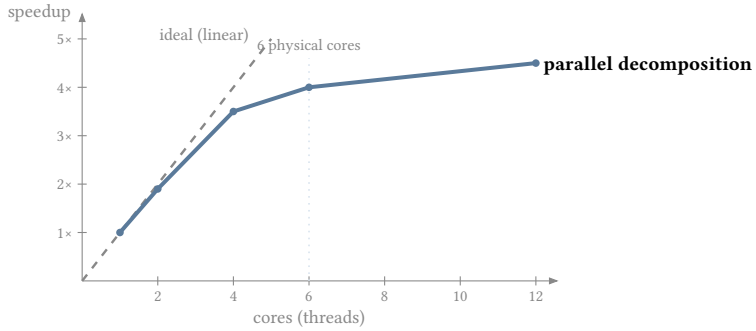

Figure 22: **The federation parallelises - measured, this engine, on one laptop.** Each scope's plan is an **independent** interior-point solve, so the scopes run concurrently in shared memory. Speedup of the parallel decomposition (blue) against core count, at **exact parity** on the plan (≥ 99.8% of demand met, the same coordination rounds either way — ∼ 30 on a low-diameter mesh, **Anderson-accelerated** where a long-diameter coupling would otherwise need one round per hop — each scope's solve **bit-identical** to the serial one). Near-perfect to the machine's 6 physical cores (1.9 × on 2, 4.0 × on 6); past that, hyperthreads and the memory-bound conjugate-gradient flatten the gain (∼ 4.5 × at 12). A datacentre node - more cores and far more memory bandwidth - scales further, and because scopes are self-contained the same code spreads **across separate machines**. Governance and computation distribute along the one seam.

7.13. What it runs on

Because the cost is **measured**, the hardware question has a plain answer - we extrapolate from numbers we actually ran. Two anchors. **Serially, on one laptop core**: a realistically-coupled national economy - ∼ 156,000 activities, an input-output table's resolution over 40 scopes and 8 periods - solves in **minutes** (the gradient-regularized engine is near-linear on a **located** mesh - measured ∼ 143,000 activities in 70 s at ∼ 490 μs/activity, flat; a **sector-rich** input-output table, dense through its commodity hubs, is heavier - the case for decomposition), at ∼ 250 bytes per technology-matrix nonzero. **In parallel**: the economy **decomposes into its scopes**, which solve concurrently - ∼ 4 × on a 6-core laptop, more on a server, and, because each scope is self-contained, **across as many machines as there are scopes**. Crucially, while the monolith now scales **near-linearly** (gradient-regularized), it is bounded by **one machine's memory**; the decomposition spreads both compute **and** memory across the federation's machines, growing only **linearly in the number of scopes** - the pull-coordination converges in a **low-diameter-independent** ∼ 30 rounds (**Anderson-accelerated** where a long-diameter coupling would otherwise need one round per hop) - so it is the decomposition, distributed, that carries the largest plans. The global economy has on the order of 10^7 to 10^8 distinct products, each using a few dozen inputs, so the technology matrix has ∼ 10^9 nonzeros (Cockshott and Cottrell's estimate) - large **sparse** linear algebra, distributed along the federation's own seams, not exotic supercomputing. Putting it together:

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in **value** and not yet consistent in **quantity** until a reconciliation step clamps each import to what its owner ships — which costs nothing in delivery here, and without which "meeting the demand in full" would describe a plan nobody could carry out.

scale	nonzeros	hardware	time
city / coarse national	$\sim 10^6$	one laptop	seconds
national, located	$\sim 10^7$	laptop (serial), multicore (parallel)	**minutes**
continental	$\sim 10^8$	a server, or a **handful of federated machines**	minutes
world, aggregated	$\sim 10^9$	a server **cluster** - one node per scope	tens of min
world, finely resolved	10^{10}-10^{11}	the federation's own machines, scope by scope	hours, fully distributed

Figure 23: **The whole-economy compute budget - extrapolated from the measured anchors, monolithic** and **decomposed.** At national scale a laptop suffices (minutes). Beyond that the monolith's **memory** footprint gives way to the **decomposition**: the plan is solved **scope by scope, in parallel, across the federation's own machines**, coordinated only by the pulls on shared goods - so the world plan is a **distributed** one, each scope solving its own corner, and the largest plans need only **more of the ordinary machines the federation already is**.

The parallelism is coarse-grained, and that is what makes it robust. The naïve idea, splitting each conjugate-gradient iteration across cores, **fails**: the iterations are too small and too many, and the synchronisation overhead swamps the work (we measured it run **slower** than serial). The granularity that pays is a **whole scope's solve per core** - each is a full interior-point problem, milliseconds to seconds of work, so the overhead vanishes against it. That is precisely the decomposition: independent scope solves, dispatched across cores in shared memory (the $\sim 4\times$ measured above) and across machines with nothing shared but the pulls on shared goods. No special hardware is required - ordinary multicore, and ordinary networking between the federation's computers. (A GPU can still accelerate each scope's sparse Hv kernel, the archetypal bandwidth-bound GPU workload; but the headline scaling is the federation, not the accelerator.)

Memory, not arithmetic, is what eventually forces distribution. At the measured ~ 250 bytes per nonzero (linear in problem size), the aggregated world plan's 10^9 nonzeros want **hundreds of gigabytes** - one fat server's RAM, comfortably; but a finely-resolved 10^{11} wants **tens of terabytes**, which no single machine holds. This is a second, independent reason to **federate** - not the political one (that comes next) but a plain physical one: past some resolution the model does not fit on one computer, so it must be solved as coupled pieces across many. The block-sparse spacetime structure makes this clean, and **we have built and measured exactly this**: each scope holds and solves its own cells, coupling only through pulls on shared resources, the scopes running in parallel and the whole converging to the same certified plan. **The optimal plan of the world economy is not a supercomputer problem.** It is a sparse convex program, decomposed along the seams of self-governance, and those scale with the ordinary, distributed hardware you give them. The barrier was never the arithmetic; it was the belief that the arithmetic could not be done.

7.14. The day under the plan: what the convex program leaves to decide

The program above decides **levels**: how much of each activity runs, in each band, at each place. Two decisions in a real day are integers, and each is decided outside the convex program and returned to it through a door the program already has. The first is **how many times** an indivisible act runs - the minimum batch and the whole count of the section before, decided on the branch-and-bound tree whose pruning the convex certificate licenses. The second is **when**, inside a band, each run begins: a timetable one person could keep - one task at a time, inside the hours they offered, on a machine that stands free, after the run that makes what the task eats. A band is the

grain of the program's time coordinate, so the program places hours in bands and the order inside one is a question of its own kind: disjunctive and combinatorial, a day of seventy tasks with no gradient to follow. So under every band of a landed plan sits a small constraint program, and the plan is the client of its answer.

The clock. The model reads a band's work and material rows as tasks: a run of a process is a task as long as the run, and inside it lie its **slices** - a person's hours on it, a means drawn for it - each a task on its holder's own clock. Time is ticks of a quarter hour from the band's start, so a day is ninety-six ticks and every start is a small finite domain, which is what lets a lazy-clause solver (Chuffed, through MiniZinc; Gecode agrees on every verdict in the suite) lay its literals eagerly. The constraints are the day's: a person's tasks are disjoint and each lies inside one of their free slots, entered by the road from home and left by the road back; two tasks of one person at two places lie the arc's hours apart; a means is drawn count-deep by a `cumulative` constraint against the units that stand at its place; a slice lies inside its run; a crew's slices start at one tick; what a process eats begins one run after its maker; a released task starts at its release, and one due at an instant ends by it. Every fact the model lays is one the plan already carries or the world states once: the row's hours, the recipe's run length, the person's window less sleep and the spans they promised in other scopes, the arc table.

Two readings of one model. Read as a satisfaction problem the question is whether the band fits, and the answer is the timetable, written back onto the plan's own rows as each Commitment's `hasBeginning` and `hasEnd`: the schedule is a read of the rows. Read relaxed the band may run over, and the model minimises the overrun - the least the band would have to be longer - and the task that ends last names the resource short, a person or a means at a place. That overrun is the band's **breach**, in the plan's own breach shape, `(resource place begin -hours)`. A solver that stops short is read once more at three times its budget and then reported as **unknown**: a verdict and a time-out are two words.

The objective is lexical, and the day keeps still. After the overrun the model minimises the drift from the plan that stood before - the starts of the last landing are the search's first guess and the drift's measure - so a band that fits keeps yesterday's starts unless something moved them, and a task that stood within two hours of now is kept still at sixteen times the weight (the freeze), so a day re-read at nine moves nobody's ten o'clock. Third and last it counts how many of a person's tasks lie outside the span they would rather work in: a want the clock weighs, never a wall.

The loop, and what Benders is here. A breach returns to the next convex solve as a **cap**: the hours the band could place on that person, or draw on that means, are what the next solve may plan there, so it reroutes the rest through storage into another band exactly as the lead prune does, and the clock reads the new plan again. Caps only tighten and are finite - one per cell and resource - so the loop ends within a few rounds, and a round limit reports the plan with what still breaches. This is the hierarchical planning pattern (Hax and Meal) closed with logic-based Benders feedback (Hooker): the band's model is the subproblem, a breach is its feasibility cut - a hard row on aggregate levels, which the convex program takes directly as a capacity bound, so slicing invariance holds - and the overrun is the linear underestimator an optimality cut would carry. The roles stand as the pattern has them with one reversal: the convex program is the loop's **client**. It keeps its barriers, its soft desires and its own branch-and-bound, receives rows, and stays the one program the plan is read off; the small models hold a band each - hundreds of operations apiece in the literature - and scale by their number. What the loop guarantees is a sequenceable

plan and the true cost of sequencing it; the certificate of the section before belongs to the convex program and stays there.

What stays outside both. Two choices a commune makes about its day are made on the rows, after the solve and before the clock. A share of a need under the scope's **block** goes to the largest share of the same need, so one hand comes for the hour and nine stay home - a block on a worker's column is a semicontinuous bound, and the tree that decides declared batches would branch over every worker, skill and cell for it. And who among the willing is asked first is the rota's word, read from what each has done and what each says of the work. The federation's word lands as rows the clock reads like any other: a crossing is a run as long as its haul, due at its departure, released at its landing.[62]

7.15. If it solves globally, why federate?

A natural objection: if the whole economy can be **solved** in one program, why keep scopes at all - why not one giant commune with one central solve? Because the objection runs together two questions the calculation debate always confused: **can the plan be computed?** - a matter of arithmetic - and **should the economy be governed from one centre?** - a matter of power. We answered the first, cheaply. The second is entirely separate, and the answer is **no**, for reasons that have nothing to do with whether the math is hard. **Centralising the computation is harmless - it is only arithmetic over what was declared in the open; centralising the authority is the road to serfdom.** The federation keeps the two apart.

The solver consumes a model it cannot produce. A global solve needs data: every recipe, every fragility curve, every ecological agent's tipping point, each scope's desires and reserve policy, the state of *this* soil and *that* machine. That knowledge is irreducibly **local and tacit** - Hayek's *real* point, the one that survives. We refuted his **tractability** argument; we never touched his **knowledge** argument, and should not. The centre can **solve** but it cannot **know**. A scope is the organ that knows its own corner and curates its own part of the model; the solver is downstream of the scopes - it eats what they grow.

The objective is constituted by the scopes, not handed down. There is no view-from-nowhere harmony to maximise. Harmony is assembled from the priority weights, the voices, the fragility profiles and reserve fractions that scopes **declare**. Who decides bread is a need never zeroed, where the soil's tipping point lies, how much insurance to hold? These are **value choices**, not computed outputs. The plan is the joint optimum *of the scopes' own declared values* - so the scopes are where the objective comes from. One giant commune means one body declaring those values for everyone: exactly the concentration we are escaping.

A central solve, run centrally, is a new central power - unless the scopes own it. If one entity runs the optimisation, that entity controls the plan, and the old planner's tyranny re-enters by the back door. What prevents it is the federation: the scopes keep ownership of **what they declare**

[62] Realized in `module/playnet/solve/sequence.scm` and `sequence.mzn`, the loop in `doors/service.scm` (`solve-current!`: up to three rounds on the solve thread, one MiniZinc subprocess per band); the requirements R1–R17 in `docs/solve/requirements.md`, the direction and its measurements in `docs/solve/direction.md`. Measured: a ten-hour band of twelve people, eight processes, an oven two deep and four processes chained by a good - 74 tasks - reads cold in 2.3 s, and a moved day in 2.5 s with the hint against 4.6 s without (Chuffed 0.13.2, MiniZinc 2.8.4, a 20 s budget); a bread town's sweep - the convex solve and its clock rounds - is nine to fourteen seconds and stays so under a district of four. Open: optimality cuts beyond the overrun, setup and changeover data, nested Benders across the federation; CP-SAT is installed and takes the hint, and is read when a second solver is wanted.

(the inputs), of the **verification** (the gap certificate lets anyone check the plan is optimal *given the declared inputs* - no black box), and of the **consent to enact** (a scope can refuse, fork, refederate - and a forked scope keeps provisioning the rest, and being provisioned by it, through the same boundary message as any peer, below). The solve is a shared instrument the federation points at itself, never a sovereign above it. And the older reasons stand untouched: **custody** (someone must be responsible for each soil, tool, and skill), **metabolic coherence** (a scope that closes its own human-time loop is resilient to disruption elsewhere - one monolith is one point of failure), and **plurality** (different scopes try different forms and may leave; a social monoculture is fragile by the federation's own logic).

Between the many, the coupling is a **pull the scopes settle at**. A federated solve runs each scope's own program and couples them only where a good crosses a boundary; the dual on the crossing is the good's pull, in labour-hours, one number for every scope the good crosses for, and it is what one more unit of one scope's ask moves in the scopes across the link. Each scope's flow on the good is signed: above nothing it supplies, below it draws, and which it does is the solve's answer - a scope is the custodian of what it holds, and whether the good crosses, and which way, is the plan's outcome. Admission names the goods that cross for a scope and nothing about their direction. Primal feasibility at convergence **is** the conservation law - what is supplied is what is drawn, the deficit of the scopes that lack the good, within what those that hold it can give - held as a test at both ends: an abundant good settles at a pull near zero with the need covered, a scarce one pins the flow to the stock's bound with the pull standing large. And the folds feed the dual before a round runs - each scope's floor-driven deficit and its stock are readable off the standing totals, so the consensus opens at the bound they state, and a run that hits its round cap names its two horns by quantity: how much must cross, and how much can.[63] What a member scope's membership **displaces** in the others is read the way a citizen's row is read: the federation solved without it, its border shut, differenced on what the others are delivered. Folds in which several goods cross at once settle as one convex solve of the same family - budgets as weights, the goods' pulls as the duals - and honesty among few, named, repeat-playing scopes is **contained rather than cured**: gains from misreporting shrink with the federation's size, and what a federated plan concedes for scope sovereignty is first-best efficiency across cells, a gap that is itself one more solve to measure.[64]

The coupling is carried by a message, and the message is the whole of what leaves a scope. A scope's half of each round is one question, answered from its own vat: given a centre per crossing good - the agreed flow less the good's scaled dual, the pull the round stands at - and the coordination weight, what signed flow does the scope's **own** program put on that good, and how full is the scope? Into a scope goes one centre per good that crosses for it; out comes a signed flow per good and a fill fraction. Recipes, columns, roster, stock, plan: none of it leaves, because the coordinator never needs it - the rounds are the single-process solve's own arithmetic, round for

[63] `warm-boundary-flows` and `federated-dilemma` (`engine/federation-solve.scm`); `tests/planner/test-boundary-conservation-law.scm`, `test-federated-dilemma.scm`. The law is load-bearing in the refine step: the final exact solve carries the proximal pull toward the agreed flow, so at convergence the augmented minimiser is the constrained one - a scope indifferent to a good it holds in plenty, under a pull near zero, supplies the need, never its whole stock. The centre enters that pull **negated** - a quadratic form costs $w(c + \sum vx)^2$, so the agreed flow z is stored as $-z$; the wire's per-scope refine, which has no cross-scope gate, is what showed a scope that draws pulled to zero when it entered unnegated.

[64] `federated-solve` (scaled-dual ADMM over the members' own programs; the boundary dual λ is the good's pull), `federated-scope-pulls` and `federated-withdrawal-displacement` (`engine/federation-solve.scm`) - the latter is `solve/externality.scm`'s `withdrawal-displacement` with a scope record for a row; `eisenberg-gale-split` for multi-good folds; the strategyproofness-in-the-large reading is Azevedo-Budish, and the second-best character of per-cell honest rules Amorós 2002, measured by `plan-welfare` (`audit.scm`).

round and pull to the ninth digit, with the scope's solve moved behind the message. So sovereignty across a boundary is a **property of the protocol**: a scope hosted in its own process, reached by a capability, settles the same economy at the same pull as one solved in place. Every reply is signed under the scope's own identity and names the round it answers, so a reply is **attributable** and answers one question once; a reply under another name, or one edited in passage beneath a valid signature, breaks the solve rather than entering it. Two consequences follow. A **fork is a peer**: two federations whose data structures and functions have diverged interoperate for as long as each answers the boundary question faithfully - the question is agnostic to the solver behind it, and the pull weighs what each asks. And the coordinator's power is bounded by the same arithmetic: it cannot speak for a scope, and dishonest centres buy it at most a coordination that fails to settle, since every scope's floors and stock are constraints of that scope's own program. A scope that misstates its own stock is **weighed, and contestable**. The pull moves against it, and it does so under its own name, where the trust substrate can see it. And its word stands beside two records it does not write alone: the flow it signs in a round is met, when the crossing settles, by a custody record the **receiving** party writes and signs, and the stock behind the flow is a count, of its instant, that any member who doubts it may contest (§Transport and storage are one recipe). A signed flow, a receipt and a count are three attributable observations of one quantity; the fold that differences them as a standing reading remains to be built.[65]

So **the global solve is the federation's instrument, not its replacement.** We made the computer one; we keep the polity many. There is a symmetry worth naming: Hayek raised two objections to planning - that it could not be **computed**, and that the knowledge is **local** so power must not be concentrated. The first we disproved, and that is what makes a coordinated plan possible at all; the second we **honour**, and that is exactly **why** the plan is federated rather than imperial. The federation is the answer to the part of Hayek that was right.

7.16. The lineage

The one solve has a lineage: every piece of it was attempted in the twentieth century, one piece at a time. Where Koopmans read the efficiency-price duality toward coordination by prices, another line set out to compute the plan itself - each attempt seizing one piece, waiting on the others, and ending politically while its mathematics held.

Kantorovich, from the 1939 factory-scheduling paper to *The Best Use of Economic Resources* (written 1942 in besieged Leningrad; publishable only in 1959): the plan as a linear program, the duals as objectively-determined valuations - transparent accounting, exactly as the pulls are read here. Its appendix already holds this program's constraint side nearly symbol for symbol:

[65] The custody legs are `transfer-custody` observations written by the receiver (`^custody-recorder`, `gateway/federate.scm`; `planner/federation/crossing-custody.scm`; `tests/planner/test-crossing-custody.scm`), and the count and its contest are `docs/observation/capture.md`. `make-boundary-oracle` and `solve-consensus` (`engine/federation-oracle.scm`) - the scope's half of the ADMM as messages, and the coordinator's loop with the per-round solves handed to the caller; `^commune-solver` and `federated-solve-wire` (`federation/wire.scm`) - the Goblins actor whose every reply is the written payload and an Ed25519 signature over it under the scope's KERI AID (`sign-bytes-under-aid`), carrying the coordinator's round id, and the coordinator that verifies before it reads. `tests/planner/test-federation-wire.scm` holds parity with `solve-decomposed` (same rounds, pull to 10^{-9}), two scopes in two vats, the scarce case settling at the conservation law's pull, and both tamperings refused; `test-federation-wire-procs.scm` hosts one scope in a **separate process** behind a websocket sturdyref and settles the economy over one near reference and one far. What a wire solve certifies is **per scope** - feasibility and the KKT gap against its own forms - with the consensus residual and the overdraw read across the flows; the coupling-graph radius needs every scope's forms in one place and belongs to the in-process solve, so a far scope whose crossing column its own program does not bound reports an unbounded gap and the certificate says so.

located-dated goods ("one and the same product… in various periods of time is treated as a different element"), methods as columns with negative entries for inputs, the plan a vector of intensities, zero-profit valuations on every method in use, transport potentials with congestion rents on the lines, and the rate of return on investment emerging as the ratio of one period's multipliers to the next - a price of time owned by no one. Two things waited. The algorithm: interior-point methods turn polynomial in 1984 (Karmarkar) - their affine-scaling form already glimpsed in 1967 by Dikin, of his own Soviet school - set later on the self-concordant-barrier footing of Nesterov and Nemirovski, the underlying Newton iteration answering to his own Newton-Kantorovich convergence theorem; and the second-order sparse solve now fits a laptop. And the objective: his optimality is undominatedness, the assortment handed down by the centre - the one missing symbol, which the scopes now declare as their own desire.

TsEMI's SOFE (from 1963): the whole economy as one optimally-functioning system, centre and sectors iterating toward a shared optimum. The ministries ground it down before it could be computed; its iteration runs here as the scope decomposition, ~ 30 pull-coordination rounds across the federation's own machines.

Dantzig-Wolfe (1960) and **Kornai-Lipták** (1965): solve the parts, coordinate through the duals, iterate - the scope solves descend from this directly. Kornai's later loss of faith was institutional - misreporting, soft budgets, gaming - precisely the hole independent verification and credit-backing close.

Glushkov's OGAS (1962-70): a nationwide network of real-time economic records, vetoed by the ministries whose monopoly on information it ended. ValueFlows event-sourcing over cheap networks builds it now as the observation layer - the easy part, four decades late.

Beer's Cybersyn (1971-73): live production signals, feedback, the operations room, destroyed in the coup. It runs here as the cascade between solves - the plan metabolised in real time.

Lange, market socialism's own author, in his last essay (written 1965, published posthumously in 1967): "Let us put the simultaneous equations on an electronic computer." The equations now solve in minutes, with a certificate.

Cockshott and Cottrell (1993): labour-time accounting, in-natura planning on sparse systems, the harmony function, the direct reply to Hayek - the immediate ancestor, completed here as one convex program over the spacetime lattice, its objective declared by the scopes, its solve federated along the seams of self-governance. The pieces assemble at the moment the missing algorithm exists and the missing politics is a federation.

8. Take back the playnet!

The animal produces within the narrow channel of its species' instinct - the beaver builds as a beaver must, the bee as a bee must. Humanity produces universally. Humans can step outside themselves and consciously adopt the functional logic of any species, any material, any context. They also know how to **listen to the object itself** - to read what the material demands, what its grain, its weight, its resistance calls for. When intention, material, and form come into genuine harmony, the result is beautiful. Humans therefore produce **according to the laws of beauty**:

the natural consequence of truly free and conscious making.[66]

Yet this capacity has been blocked by two structural conditions that industrial capitalism inherited and intensified: the **private ownership** of the means of production and consumption, and the **division of labour**. The first compels: when the tools, materials, and means of life are privately held, one enters production to survive. The second confines: when labour is fragmented into narrow, repeated operations, the worker is locked - like the animal - into a single channel. Together they close off both the freedom of perspective and the attentiveness to material that the laws of beauty require - as a structural consequence of how production is organised and owned.

If these two conditions produce the estrangement, then undoing them must undo it. The commons places the **means of production** in common ownership. The **means of consumption** flow through two rails - individual claimable and communal - satisfied in harmony. Every matched hour grows the citizen's consumer voice v_p^{cons} - simultaneously accruing producer voice (how do we make it?), consumer voice (what does the community need?), and drawable credit (what do I get?).

The commons dissolves the **division of labour** through several interlocking mechanisms. Producers freely associate across any scopes in all branches of human activity, fulfilling work-intents wherever the plan calls for their labour, joining and leaving as the work and their own development require. What is organised is the **production**; the producers move freely. Within each plan, **who does which part** is determined by equal citizenship weight - preferences over slots, shifts, and partners carry the same say regardless of past contribution. The closed metabolic loops collapse the divisions between planning and doing, between production and consumption, into the same association - the people who plan the loop, produce within it, and consume its output are, by structural necessity, the same producers. And as f rises, free time provides the ground for the **all-around development of the individual** - cross-skilling, rotation, access to developmental work become the norm. The citizenship share absorbs the whole consumer voice; the labour-measure recedes because **the social-material ground that required rationing has faded into abundance**. The antithesis between mental and physical labour fades; the relation among producers becomes the **free and equal association** itself; and the laws of beauty actually proceed.

Any branch of activity - food production, care, education, construction - whose participants begin to form a social plan with the means around them is, **from that moment**, a living scope in the Universal Commune. Workers, professionals, students, carers, retirees - the mechanism is universal. Anyone can offer availability, be matched, and begin. The private plans that currently organise those same means need only be **outgrown**. When the common plan works better - when it actually serves the producers as a whole - people follow it. **Every branch of activity is a site where the common plan and the private plan compete for the same hours**, and the common plan carries the structural advantage: it returns voice over the organisation of their own activity to the people who actually do the work.

8.1. Liberate Humanity from Nature and Nature from Inhumanity!

Humanity organises its productive activity through **knowledge**: recipes, resource specifications, plans - composable, searchable, encoded in the three VF layers. Natural systems produce in

[66] Marx, *Economic and Philosophic Manuscripts of 1844*, "Estranged Labour": "an animal forms things in accordance with the standard and the need of the species to which it belongs, whilst man knows how to produce in accordance with the standard of every species, and knows how to apply everywhere the inherent standard to the object. Man therefore also forms things in accordance with the laws of beauty."

accordance with **nature's own dynamics**: an ocean, a forest, a watershed, a soil microbiome each self-organises through its own dynamics. Their outputs are use-values: fishery yields, soil fertility, water purification, carbon sequestration - all reducing the total labour-time the federation must expend.[67]

An **ecological agent** (VF) - an ocean, a forest, an estuary - is a natural system recognised as an agent that produces through its own dynamics. Every scope that draws from natural systems holds **custody** of the ecological agents it interacts with, and carries **ecological specifications** in its plan: regenerative envelopes, absorptive capacities, sustainable yields. A food-production scope stewards its soil; a fishing scope stewards its fishery; a forestry scope stewards its forest. The VF **contained-in** relationship models the nested structure of ecosystems - nutrient flows, material cycling, ecological containment. A single scope can steward many ecological agents: a coastal scope might steward the fishery, the estuary, and the mangrove.

Stewardship is administration applied to natural systems. The question is the same: does the intent match the specification? At the scope level, the producers who work the land verify the soil; the producers who fish verify the catch - as part of the same self-governance that verifies any other specification. At the UC level, independent verifiers check that the draw respects the regenerative envelope. The ecological specification is verified by the same mechanism that verifies a loaf of bread or a surgical procedure.

The products of nature, once drawn in by the scope's plan, enter the total social product along with their mode of distribution: consumed in production (means of production), allocated as communal consumption, or distributed as individual-claimable goods. The ecological agent's yields are denominated in natural units (fish stocks, water quality, soil fertility), recorded as economic events with the ecological agent as provider. A fish caught from a stewarded fishery and a loaf baked from a recipe both appear as area on the surface - the direct labour $\mathrm{SNLT}_{\mathrm{direct}}$ of catching or baking - and both carry an embodied cost $\mathrm{SNLT}_{\mathrm{crystallized}}$ for credit. One is human production through knowledge; the other is nature's gift, made available through the scope's stewardship.

The general pattern is this: wherever humanity understands a natural process well enough to regulate it, that process can be inserted into social production as a means - and the direct human labour shrinks to the role of watchman and regulator.[68] Agriculture becomes the application of the science of material metabolism - soil chemistry, nutrient cycling, microbial ecology - regulated for the greatest advantage of the whole social body. The farmer steps to the side; the process of nature, understood and guided, does the producing. Aquaculture follows the same pattern: understanding water chemistry, breeding cycles, and trophic flows reduces human labour to monitoring and

[67] Marx, *Grundrisse* (Notebook VII, "Fragment on Machines"): "Labour no longer appears so much to be included within the production process; rather, the human being comes to relate more as watchman and regulator to the production process itself. No longer does the worker insert a modified natural thing as middle link between the object and himself; rather, he inserts the process of nature, transformed into an industrial process, as a means between himself and inorganic nature, mastering it. He steps to the side of the production process instead of being its chief actor. In this transformation, it is neither the direct human labour he himself performs, nor the time during which he works, but rather the appropriation of his own general productive power, his understanding of nature and his mastery over it by virtue of his presence as a social body - it is, in a word, the development of the social individual which appears as the great foundation-stone of production and of wealth."

[68] Marx, *Grundrisse* (Notebook VII): "But to the degree that large industry develops, the creation of real wealth comes to depend less on labour time and on the amount of labour employed than on the power of the agencies set in motion during labour time, whose 'powerful effectiveness' is itself in turn out of all proportion to the direct labour time spent on their production, but depends rather on the general state of science and on the progress of technology, or the application of this science to production. Agriculture, e.g., becomes merely the application of the science of material metabolism, its regulation for the greatest advantage of the entire body of society."

maintenance while the ecosystem yields fish, shellfish, and seaweed. Forestry becomes the science of succession and canopy dynamics - planting, thinning, and harvesting timed to the forest's own growth cycles. Energy production harnesses solar radiation, wind patterns, and hydrological cycles - natural processes transformed into industrial processes, with human labour confined to installation and upkeep. Bioremediation turns microbial metabolism into waste processing. Composting turns decomposition into soil production.

In each case the pattern is the same: science reveals the natural dynamic; the scope regulates it for the advantage of the whole social body; administration verifies the draw against the ecological specification; and f rises - because the **free gifts of nature**, once understood, replace direct human labour on a scale **out of all proportion** to the effort invested. The monstrous disproportion between the labour-time applied and its product - which under capital appears as the crisis of value - here appears as what it truly is: **the liberation of time itself**.

The social individual, understanding nature and mastering it by virtue of their presence as a **social body**, steps to the side of the production process, and the **development of that individual** becomes the great foundation-stone of production and of wealth. Every river harnessed, every soil understood, every cycle regulated for the common good returns to the federation as **time** - time for science, for art, for care, for play, for the **all-around development of every person**.

The earth itself becomes a **partner** in the association - and in that partnership the ancient separation dissolves. Natural science, by revealing the dynamics that sustain human life, becomes the basis of human science; the science of humanity, by organising social production around those dynamics, incorporates natural science into itself. This convergence is not new - industry already accomplished it on every factory floor where chemistry became production, every field where biology became agriculture - but accomplished it in an **estranged form**, under the blind compulsion of value. What is new is the **conscious form**: the recipe that encodes a fermentation, the plan that coordinates a harvest around the regenerative capacity of the soil, the specification that translates a watershed's cycles into a limit the community can see and govern. In each, **understanding nature and organising human life are no longer two activities but one**. There will be one science.[69] As the federation learns to produce in accordance with nature, the necessary labour of society shrinks to a minimum, and what remains is the free development of individualities - with the means created, for all of them.

And so the measure gives way. Economy of time is the first economic law of communal production - but it is the first law of the **realm of necessity**, and its dominion reaches exactly as far as necessity's.[70] We understand time, socialise it, distribute it consciously across the branches - we **seize all time** - in order that we may finally **cease to be measured by it**. For as the free gifts

[69] Marx, *Economic and Philosophic Manuscripts of 1844*, "Private Property and Communism": "Industry is the actual, historical relationship of nature, and therefore of natural science, to man. If, therefore, industry is conceived as the exoteric revelation of man's essential powers, we also gain an understanding of the human essence of nature or the natural essence of man. In consequence, natural science will lose its abstractly material - or rather, its idealistic - tendency, and will become the basis of human science, as it has already become - albeit in an estranged form - the basis of actual human life, and to assume one basis for life and a different basis for science is as a matter of course a lie. … Sense-perception must be the basis of all science. Only when it proceeds from sense-perception in the two-fold form of sensuous consciousness and sensuous need - is it true science. All history is the history of preparing and developing 'man' to become the object of sensuous consciousness, and turning the requirements of 'man as man' into his needs. History itself is a real part of natural history - of nature developing into man. Natural science will in time incorporate into itself the science of man, just as the science of man will incorporate into itself natural science: there will be one science."

[70] The first law of communal production governs the realm of necessity; its success contracts that realm, and with it the reign of the measure.

of nature replace direct labour and L falls toward its watchman's residue, $f \to 1$: the domain over which labour-time rules contracts toward a point, and labour time ceases and must cease to be the measure of the wealth produced. The surface audits its own obsolescence - it is the ledger of unfreedom, keeping its accounts in order to retire what it accounts.

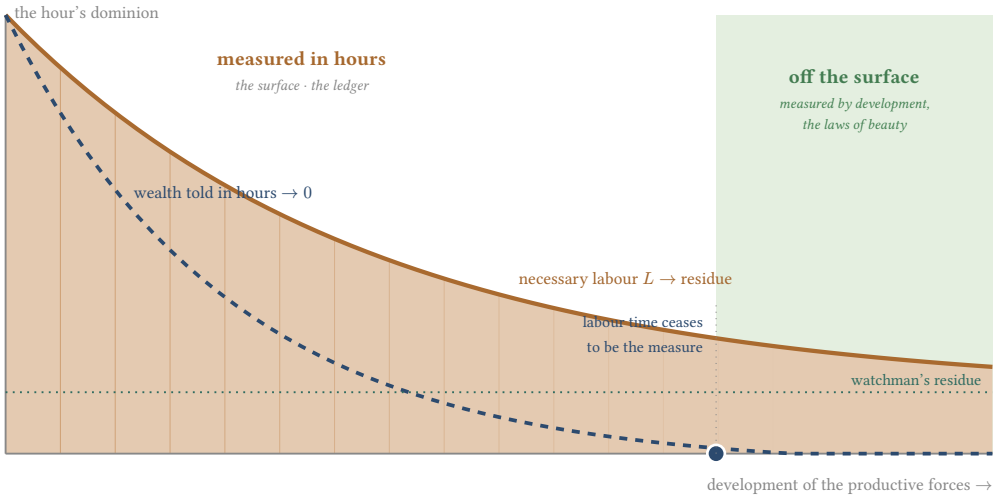

Figure 24: **The measure withers.** As the productive forces develop (rightward), necessary labour L falls toward a watchman's residue - never quite reaching zero, since some necessary labour always remains. But the **hour's dominion** contracts faster: the share of wealth still **told in hours** falls all the way to zero, and past that point "labour time ceases and must cease to be its measure." The amber ledger - the surface, reckoned in hours - dissolves rightward, and what opens beyond it is **off the surface** entirely: its measure no longer the quantitative, imposed hour but the qualitative, self-authored development of individuals - production according to the laws of beauty.

What opens above the residue is not **more** time, told in the same hours. Free activity has no cell on the surface; at the limit it ceases to be time-denominated at all. Its measure passes from the **quantitative and imposed** - the socially necessary hour - to the **qualitative and self-authored**: the all-around development of the individual, production according to the **laws of beauty**, the free association itself. Time stops being our measure so that individuals may become their own. This is the higher law that supersedes the first: the economy of time is the highest law of the realm of necessity, and its full triumph is the withering of its own dominion.

Workers & players of the world, unite!

It's time to take back the playnet!

$$\underbrace{p}_{\text{a citizen}} \quad \underbrace{N}_{\text{total citizens}} \quad \underbrace{T}_{\text{hours per citizen}} \quad \underbrace{s}_{\text{a scope (branch of activity)}} \quad \underbrace{d}_{\text{a desire (bread, housing, care...)}} \quad \underbrace{r}_{\text{a resource}} \quad \underbrace{\mathrm{SNLT}_{\mathrm{direct}(r)}}_{\text{direct labour per unit}} \quad \underbrace{\mathrm{SNLT}_{\mathrm{crystallized}(r)}}_{\text{crystallized labour-cost}}$$

$$\underbrace{L_p}_{\text{a citizen's matched hours}} \quad \underbrace{L_{p,s}}_{\text{hours in a scope}} \quad \underbrace{\sigma_p}_{\text{a citizen's satisfaction}} \quad \underbrace{g_d}_{\text{gap on a desire}}$$

$$\underbrace{L}_{\text{total labour}} = \overbrace{\sum_{p}}^{\text{add up over all citizens}} L_p \qquad \underbrace{L_s}_{\text{scope total}} = \sum_{p} L_{p,s} \qquad \underbrace{f}_{\text{free-time}} = 1 - \overbrace{\frac{\Lambda}{N \underbrace{\cdot}_{\text{times}} T}}^{\text{the plan's planned hours divided by citizens times hours}}$$

$$\underbrace{v^{\mathrm{prod}}_{p,s}}_{\text{producer voice}} = \frac{L_{p,s}}{L_s}$$

$$\underbrace{a_p \in [0,1]}_{\text{capacity — a governed assessment}} \qquad \underbrace{w}_{\text{aggregate incapacity}} = \frac{\sum_q (1 - a_q)}{\sum_q 1} \qquad \underbrace{l}_{\text{labour share}} = 1 - f$$

a_p is a **governance assessment**: set by a sealed call and issued as a signed credential carrying its assessor, timestamp and reason, so a capacity judgement is attributable and contestable by the ordinary governance route. The standing design intent is to derive it as the labour-weighted average of other citizens' attestations, $\sum_{q \neq p} L_q x^{\mathrm{cap}}_{q,p} / \sum_{q \neq p} L_q$, — capacity-recognition as a production-governance act, self-attestation excluded, weighted by producer voice — and that aggregation **remains to be built**. The mean in w runs over account-scopes rather than as $1/N$ because a citizen active in several scopes is counted once per scope; the two agree exactly when each citizen holds one account.

$$\underbrace{v^{\mathrm{cons}}_p}_{\text{consumer voice}} = \underbrace{l(1-w) \cdot \frac{L_p}{L}}_{\text{contribution}} + \underbrace{lw \cdot \frac{1 - a_p}{\sum_q (1 - a_q)}}_{\text{solidarity}} + \underbrace{\frac{f}{N}}_{\text{citizenship}}$$

$$l(1-w) + lw + f = 1 \qquad f \overbrace{\to}^{\text{approaches}} 1 \overbrace{\Longrightarrow}^{\text{implies}} v^{\mathrm{cons}}_p \to 1/N \quad \overbrace{\forall p}^{\text{for every citizen}}$$

$$\mathrm{SNLT}_{\mathrm{crystallized}(r)} = \mathrm{SNLT}_{\mathrm{direct}(r)} + \sum_x a_{xr} \cdot \mathrm{SNLT}_{\mathrm{crystallized}(x)}$$

one recursive value rollup, applied to every resource alike — a_{xr} is the amount of x in one r: quantity of a material consumed, wear δ of a means worn (depreciation), or amount of a substance contained

$$\underbrace{\mathrm{area}(r) = R_r = q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}}_{\text{surface: direct labour}} \qquad \underbrace{L = \overbrace{\sum_r}^{\text{all cells}} q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}}_{\text{total social labour, each hour once}}$$

$$\underbrace{K = \overbrace{\sum_{r \in \mathcal{R}}}^{\text{claimable set}} \max(0, q_r - s_r) \cdot \mathrm{SNLT}_{\mathrm{crystallized}(r)}}_{\text{credit pool: embodied cost}} \qquad \underbrace{W \cdot v^{\mathrm{cons}}_p - d_p}_{\text{drawable credit}}$$

$$R_r = \underbrace{\text{“maintain“}}_{\text{hold target}} + \underbrace{\text{“expand“}}_{\text{raise target}} + \underbrace{\text{“insure“}}_{\text{reserve}} \qquad \underbrace{q^*_r}_{\text{desired · soft}} + \underbrace{q^{\mathrm{res}}_r}_{\text{insurance · hard}} = \underbrace{\text{target}}_{\text{full standing level}}$$

$$\underbrace{\hat{x}_p(d)}_{\text{priority weight on a desire}} = \overbrace{\prod_{i \in \mathrm{path}(d)}}^{\text{multiply along the path}} \underbrace{w_{p,i}}_{\text{weight at each branch}}$$

$$\underbrace{w_d^{\text{desire}}}_{\text{social desire}} = \sum_p v_p^{\text{cons}} \cdot \hat{x}_p(d)$$

$$\underbrace{\text{Score}}_{\text{harmony}} = -\overbrace{\sum_d}^{\text{add up over all desires}} w_d^{\text{desire}} \cdot g_d^2$$

the shared-dimension member; a citizen's **own** stated want rides the bounded **cap** member beside it - $w/(1+s/c)$, the ask a wall - so that stating it honestly is the citizen's best play (*Honest by shape*)

$$\underbrace{M(s)}_{\text{metabolic coherence}} = \frac{\overbrace{F_{\text{internal}(s)}}^{\text{flows within scope}}}{\underbrace{F_{\text{total}(s)}}_{\text{all flows}}}$$

$$\underbrace{\Phi}_{\text{social desire satisfaction}} = \frac{N}{\sum_p 1/\sigma_p} \qquad \underbrace{\mathcal{P}}_{\text{social play time}} = \overbrace{\min}^{\text{the smaller of}}(f, \Phi)$$

Both are **measures of the plan**: the solver maximises harmony, and these read the result off it. The engine reports `meanfill` — the **arithmetic** mean over **desires** of $\min(1, s/D)$ — and that is the figure behind every "met %" in this document. Φ is the stronger statistic: a **harmonic** mean over **citizens**, so it collapses when a single term approaches zero and it registers an individual left behind. Reporting it calls for per-citizen σ_p, which calls in turn for desires attributed to citizens; that attribution, and $\mathcal{P}$ with it, **remains to be built**.

The planner, as implemented

the convex program over activity levels, its primal-dual central path & gap certificate, recipe choice, and the sparse spacetime matrix it all rides

$$\underbrace{g_d}_{\text{gap}} = \max\left(0, \frac{q_d^* - q_d}{q_d^*}\right) \qquad \underbrace{w_d}_{\text{emphasis mass}} = W \sum_p v_p \hat{x}_p(d) \;\; (W = 1000 \text{ , one whole hand}) \qquad \underbrace{\text{desire-tension}}_{\text{soft, bounded}} = w_d \cdot g_d^2$$

$$\underbrace{\varphi(s)}_{\text{a fragility's tension — one profile per KIND; the registry supplies } \varphi,\ \varphi',\ \varphi''} \qquad \text{canonical kind: } \underbrace{B_t(u,c) = -t\log(c-u)}_{\text{the interior-point log-barrier}} \quad \text{on} \quad \underbrace{L_k \le A_k \ ,\ D_r \le C_r \ , \ldots}_{\text{skill, ecology, \& each governance-set profile}}$$

$$\underbrace{h}_{\text{harmony}} = -\left[\overbrace{\sum_d w_d g_d^2}^{\text{unmet desire}} + \overbrace{\sum_k B_t(L_k, A_k)}^{\text{scarce labour}} + \overbrace{\sum_a B_{w_a}(\cdot)}^{\text{fragility: FIXED weight}} + \overbrace{\sum_n \varphi_n^{\text{held}}}^{\text{floors that hold}} + \overbrace{\sum_p w_p/(1 + s_p/c_p)}^{\text{own wants: cap}} + \overbrace{\sum_c w_c(\cdot)^2}^{\text{congestion}} \right]$$

Six families, not two, and the weight is what separates them. The **swept** weight $t \to t_{\min}$ binds only **at** the wall — right for an accounting identity, since all the flour should be used. A **fixed** governance weight w_a is felt **at the optimum**, which is what lets the federation ease off **before** a limit rather than halt at one. φ^{held} diverges at bare need and **satiates** at a margin, so scarcity erodes the margin and never the floor. The **cap** is a citizen's own want: bounded, the asked quantity a wall among the barriers rather than a number in the value, so any rewrite of one's own asks is weakly dominated (*Honest by shape*). Congestion is two-sided and quadratic. Only the first is a B_t; writing harmony with B_t alone hides the distinction the ecological and reproduction floors depend on.

$$\underbrace{\hat{\ell}(i,k)}_{\text{skill-}k\text{ labour embodied per unit of } i} = \ell(i,k) + \overbrace{\sum_{j \in \text{BOM}(i)} a_{ij}\hat{\ell}(j,k)}^{\text{active-set form zeroes branches through stock-covered inputs}}$$

$$\underbrace{\lambda_k}_{\text{skill pull (dual)}} = \frac{\partial B_t}{\partial L_k} = \frac{t}{A_k - L_k} \qquad \underbrace{\text{pull}(i)}_{\text{loudness}} = \overbrace{\frac{2w_d g_d}{q_d^*}}^{\text{desire}} - \sum_k \hat{\ell}(i,k)\lambda_k - \sum_r \hat{d}(i,r)\mu_r$$

$$q \leftarrow (q + \eta \cdot H^{-1}\ \text{pull})^+ \qquad \text{concave} \quad \Rightarrow \quad \text{global optimum; halts as gap} \quad \mu \to t_{\min}$$

$$\underbrace{H = \text{diag}\left(2w_d/(q_d^*)^2\right)}_{\text{desire curvature — diagonal because the variables here are the finals } q \text{ (REDUCED program, intermediates rolled into } \hat{\ell})} + \underbrace{\sum_k B_k'' \hat{\ell}_k \hat{\ell}_k^\top + \sum_r B_r'' \hat{d}_r \hat{d}_r^\top}_{\text{crystallised coupling: finals sharing a scarce skill or cap}} \qquad B'' = t/(\text{cap} - \text{use})^2$$

$$\underbrace{q^*(t) = \text{argmax}_q h_t}_{\text{central point}} \qquad \underbrace{t \to t_{\min}}_{\text{follow the path: analytic centre} \to \text{optimum — to a FLOOR, not to 0}} \qquad \underbrace{\lambda_i \cdot \text{slack}_i = t}_{\text{KKT in the limit}}$$

$$\underbrace{f(x) - f(x^*) \le m \cdot t + \min(\|r\| \cdot D, \|r\|^2/(2\sigma))}_{\text{the WHOLE inequality: } m{\cdot}t \text{ alone bounds nothing unless stationarity is exact}}$$

Two **independent** bounds on the distance term, so the smaller is sound and they fail in different places. *Geometry* $\|r\| \cdot D$ needs a bounded feasible set, and gives ∞ the moment one variable is uncapped. *Curvature* $\|r\|^2/(2\sigma)$, from σ-strong convexity, needs no geometry at all — so it still bounds an **unbounded** program, and gives ∞ only where the objective is flat. The second is not a refinement: a federated scope's crossing column is a free source that nothing in the scope's own matrix bounds, so the geometry branch is dead exactly where federation lives. The same σ sizes the **honesty radius** $2W/\sigma$ - how far any rewrite of one citizen's asks can move the plan - so one modulus certifies both the optimum's margin and the mechanism's manipulation bound.

$$\underbrace{x_b \in \{0\} \cup [L_b, \infty)}_{\text{run at scale or not at all — non-convex because the WORLD is}} \qquad \underbrace{\text{node bound} = f(x) - \text{gap}(x)}_{\text{the convex certificate LICENSES the pruning}}$$

$$\underbrace{\text{branch \& bound}}_{\text{exhausted tree} \to \text{certified global}} \qquad \underbrace{\text{nodes} \sim 1.5B \quad \text{vs} \quad 2^B}_{\text{measured: } B{=}12 \text{ visits 19 of 4096, 99.5\% pruned}}$$

Which recipe makes a good is **not** here: two mills are two columns and the plan may run both, so technique choice is **inside** the convex program and comes out as a level, not a selection.

$$\underbrace{y_g = \text{init}_g + \sum_a T_{ga} x_a}_{\text{located-dated balance — production, transport (space), storage (time, forward) are all COLUMNS } a}$$

$$\underbrace{H v = \sum_\ell B''_\ell \, a_\ell \left(a_\ell{}^\top v\right)}_{\text{LOCATED program: variables are the activities } x\text{, so EVERY tension (desire too) is one rank-1 term — matrix-free, } O(\text{nnz})\text{/step, near-linear on a chain}}$$

$$\underbrace{\max_{x_s} h_s - \sum_b \frac{\rho}{2}\left(x_b - (z_b - u_b)\right)^2}_{\text{a SPATIAL mesh is harder: split by SCOPE } s\text{, each an independent solve run in PARALLEL}}$$

$$\underbrace{z_{bp} = y_{bp} - \overline{y}_b \;,\quad \sum_p z_{bp} = 0 \;,\quad \lambda_b = \rho u_b}_{\text{boundary good } b\text{: each scope's SIGNED flow } y_{bp}\text{, supplied above nothing and drawn below; the flows sum to nothing, coordinated by its PULL}}$$

Rounds scale with the coupling **diameter**, not with the number of scopes: at tolerance 10^{-4} a 16-scope **star** converges in 35 rounds, a 16-scope **chain** in 105, and a 32-scope chain in 247. A pull has to walk the graph, and a chain is where it walks furthest — the same quantity that governs how far the federated certificate sits from the monolithic one.

$$\min_x \overbrace{\sum_{w,j} x_{wj}\left(1 + \tau \cdot \text{km} + \pi_{w,s}\right)}^{\text{proximity \& preference, IN the free-time coefficient}} + \overbrace{\sum_w \frac{\rho}{h_w H} L_w{}^2}^{\text{convex per-worker load } \propto (L_w/h_w)^2 \text{ — FRACTIONAL: scale-free, bounded}}$$

equal marginal tension $\longrightarrow$ EMP: equal share of each one's offered time

Not solved separately. A worker is a supplier bundle — one barrier capping their offered hours, one congestion term above — carried in the **same** program as everything else, so **who works which slot is the worker-column level of the ordinary solve** and inherits the ordinary certificate. A standalone matcher would have to report its own shortfalls; here a roster that cannot staff the plan is simply **infeasible**, with the violation quantified. Proximity is not a separate objective either: distance and skill-time preference ride **inside** the worker column's free-time coefficient, so a distant or reluctant hour simply costs more of the thing the plan is already economising. The preference $\pi_{w,s}$ is read clamped to $[-\beta, \beta]$ and centred on the worker's own mean over their skills, in the congestion's unit $2\rho/H$: equal for every citizen, blind to hours.

$$\underbrace{\ell'_j = \ell_j \cdot \left(1 - \tau_v \cdot s_j\right)}_{\text{the work column's hours, re-weighed}} \qquad \underbrace{s_j = \text{clamp}\left(P_j - \sum_g \text{into}(j,g)\lambda_g, -1, 1\right)}_{\text{the push of the parts covering } j\text{, centred on its good}} \qquad \underbrace{0 \le \tau_v < 1}_{\text{governed; default 0}}$$

The same form as proximity and preference, on the other rail: a bounded multiplier on hours, in the coefficient the plan already economises. P_j sums, over the parts covering j, each part's voice-weighted stances - a citizen's weight on a part their share of the validated hours on it, their hand held hour by hour so each hour speaks once. The λ_g solve one goods-by-goods system before the solve so that every good's felt hours equal its clock hours at the worked mix: a statement about **what** cancels, and only the choice among a good's **ways** moves. Every coefficient stays within $[(1-\tau_v), (1+\tau_v)]$ of its clock hours and positive; walls, floors and decided columns are untouched; the books keep clock hours. τ_v is the field's own governed number, distinct from the transport τ above.

*Read as a whole, this convex program is the federation's **active inference**: its Markov blanket, with harmony as variational free energy and the observer's democratic recognition as the weight on evidence - developed in the companion* planner-as-active-inference.

9. Technical appendix: the credential substrate

The observer is a fold over an immutable log of economic events. Everything it reports - who holds which skill, how many hours someone has practised, what credit they have earned and drawn - is **derived** from that log, never stored as a mutable field, so nothing can drift and a retroactively-voided event simply drops from every total. This appendix records how that log is made **portable**: each event becomes a signed, edge-chained credential, anchored to a self-certifying identifier, so any party can replay the fold from the records alone rather than trusting one server's state.

9.1. Skills are conformance, derived from the work log

A skill is a resource specification; a person **has** one when they hold an economic resource that conforms to it, and a recipe's `work` flow names the skill it needs the same way. The single rule for "may this person do this work" is **conformance**: their held skill conforms to the required specification **directly**, or **through a substitute**. A specification carries a `satisfied-by` relation - a disjunction of skill bundles, each component with a minimum practised-hours threshold - so one person holding electrician and surgeon at sufficient hours conforms to an electrosurgery requirement without ever holding "electrosurgeon" directly. The planner reads this as a set **expansion**: a worker's effective skills are the transitive closure of what their held skills satisfy. Composition is therefore not a new matching mechanism but the planner's existing "supply only the skills you hold" applied to an expanded set - the x-vector solve is unchanged, and a substitute still draws on the one shared hours budget, so nothing is double-counted. Eligibility is thus a **relation a body satisfies today**, kept by practice, not a fixed title: no one **is** an electrosurgeon - a bundle of practised skills **conforms** to the requirement, and the conformance closure is recomputed each solve against a living practised-hours ledger.

There is no certification flag. The distinction between a skill learned by **doing** - where mistakes are reversible - and one that must be **trained before** doing - where they are not - is emergent: a skill reachable by substitution from commonly-held skills is learned on the job, while one whose only path is a dedicated training recipe is gated by that recipe's existence. Hours are derived too: a person's competence in a skill is the sum, over their work events conforming to it, of validated labour-time - the **same** quantity the credit ledger sums, so the two can never disagree.

9.2. One event, one credential

Each kind of planner event becomes its own credential, and the references between them become cryptographic **edges**, so the history is a single walkable graph:

Work-event - issued when a person performs work; edges to the commitment it fulfils, the intent it satisfies (the validation anchor), and - after the first - the **skill** it exercises.

Skill - the membership token, minted **once** per (person, skill) on first performance, edged to its founding work event. It carries no hours; competence is summed from the work log.

Intent and **commitment** - attest the **desired** quantities, which the credit gate needs to compute how fully an intent was met, and which let the still-open part of a plan be reconstructed.

Claim - the consumption mirror of the work-event credential: issued when a person draws earned credit to receive a good, recording the labour-hours spent.

9.3. The whole ledger reconstructs from the chain

Because the observer is a fold and the log is portable, every view it computes can be **re-derived** from a held set of credentials: the **skills** a person holds are their skill credentials; their **practised hours** in a skill are the work-event hours reached through its edges; and their **credit balance** is gross - the hours of their work-event credentials - minus claimed - the hours of their claim credentials. Contribution and consumption are thus symmetric halves of one ledger on one event log, and the **full** balance, not merely the earnings side, is reconstructable offline and verifiable against the issuer's signatures. Revocation needs no reconciliation: dropping a credential removes one node, and every total recomputes.

Cryptographic chains prove what **exists**, not that nothing is **missing**. A periodic **derived-state snapshot** closes that gap - a signed enumeration of the expected set, edging to every credential it should contain, so a holder can confirm completeness rather than assume it. The records are identity-anchored throughout: each is signed under the issuer's autonomic identifier, so competence and contribution travel with the person - offline-checkable, across federations, without exposing who they are - and the in-memory credential store the planner uses is interface-identical to the live, witnessed one, so the same substrate runs in a test and in the field.

10. Appendix: Why the objective can be trusted

Two measurements on the harmony objective: that its score tracks equal satisfaction and parts from it only at a floor the federation chose, and that stating a want honestly is each citizen's best play.

10.1. Equal satisfaction and the floor

With each citizen's satisfaction σ_p, its harmonic mean Φ, and the social play time $\mathcal{P}$ as defined in *Labour, skills, and free time*:

$$\underbrace{\sigma_p}_{\text{a citizen's satisfaction}} \in [0,1]$$

$$\underbrace{\Phi}_{\text{social desire satisfaction}} = \frac{N}{\sum_p 1/\sigma_p}$$

$$\underbrace{\mathcal{P}}_{\text{social play time}} = \min\left(\underbrace{f}_{\text{free-time}}, \underbrace{\Phi}_{\text{equal satisfaction}}\right) \in [0,1]$$

The sensitivity to whoever is worst off is the whole reason for the harmonic mean, and it is why the figure the engine reports today is **not** this one: `meanfill` returns the arithmetic mean over **desires**, which neither sees an individual nor collapses when one term vanishes. Attributing desires to citizens - and with it Φ and social play time as reported quantities - remains open.

The solver turns its crank on harmony, and social play time is why the crank was built. The two are joined at two points, and both joints are visible in the grammar of the tensions.

Free time is carried by **satiation**. A desire-tension is $w_d g_d^2$ while a want is unmet and **zero** once it is met, so harmony cannot be bought by making more than anyone asked for. Labour is bounded above by what the demands require, and the free-time term κL prunes what is left.

Equal satisfaction is carried by the **shape** of the penalty. The quadratic shortfall harmony relieves and the harmonic mean Φ rewards are both convex and falling in satisfaction, and both are spent against the same labour budget, so their optima very nearly coincide. On a two-need economy where one need costs four times the other, harmony under uniform weights settles at 85.3% and 41.2%; the split that maximises Φ outright is 83.3% and 41.7%. Harmony reaches $\Phi = 0.5554$ where the best available is 0.5556. Serving the cheaper need further is what a harmonic mean asks for — it buys satisfaction where satisfaction is cheap, and stays sensitive to the worst-off precisely because the last units bought are the dearest.

The two measures part company at the **bottom** of the scale, and the federation's answer is already in the grammar. A desire-tension is **bounded**: a wholly unmet want costs w_d and no more, so a want priced far above the rest can be set aside for a finite price. Φ has no such ceiling — a satisfaction approaching zero carries the harmonic mean to zero with it. So the tensions that must not be traded away are not written as desires at all. A **reproduction floor** is a possibility-tension: a barrier, divergent as its slack closes, which no weight on any desire can outbid. Measured on that same economy with the dear need priced at a hundred: left as a **want** it settles at 1.5% and Φ falls to 0.030; declared a **floor** it is met exactly, and the cheaper need pays for it.

So the coupling holds **unconditionally** beneath the floors, where it is not an optimisation but a feasibility, and holds **closely** above them, where harmony and the harmonic mean want nearly the same thing. What a federation decides is where the floor lies — which tensions are possibilities and which are desires. That decision is political, and the arithmetic is deliberately unable to make it.

10.2. Honest by shape, in full

The desire family has two members, and **who states the want** chooses between them. A dimension the scope holds in common - a sub-commune's folded total, a row a group co-signs - carries the fractional-gap quadratic of *Desire as harmony*, and the assortment scales together. A want a citizen states **for themselves** enters as the **cap** member: its value is bounded and never reads the asked quantity - $\varphi = w/(1 + s/c)$, with c the spec's reference scale, a catalogue fact - and the quantity stands beside it as a wall, $s \leq q$. The pull is the citizen's weight the ask is logistics. At the optimum every such want is served to the lesser of its ask and its **water level** - a level set by the citizen's **conserved** weight against the good's scarcity, never by the ask itself: the report that moves the level is the citizen's one hand of emphasis, bounded to one whole hand and shared with every other row it attaches to (§Desire as harmony), not the unbounded quantity a bare ask could otherwise state.[71]

What the shape buys is proved, and machine-checked. Any rewrite of a citizen's own asks - padding, shading, a decoy row, any mixture across any cells drawing on a shared supply - is weakly dominated by stating them as they are; the per-solve obligations are exactly two, that the deviation's solve respects supply and that the truthful solve wastes nothing. Across goods converted by real technology, no rewrite obtains a bundle worth more at the **truthful** solve's own technology-anchored pulls - a lie slides the citizen's bundle along the isovalue surface and never climbs it (the deviant solve's own duals move with the lie, and are not the pulls this holds at). The rule is **obviously** strategy-proof - a member of the sequential allotment family, honest even for a citizen who holds no model of anyone else. And the ask's own wall is never the exact inequality the theorems above assume it to be - the engine enforces it as a swept barrier whose weight is floored, never zero - yet the softness buys nothing: satiation caps a citizen's own utility at their peak regardless of how much slack the wall hands out past it, and the shared level, reading only the **other** reports, caps it below regardless of how much slack their own smaller ask bought. The one channel a soft wall could still open - the shared level itself drifting with a softened report through the supply-balance equation - is not a new incentive question; it is exactly the same certified tolerance every dominance claim above already carries.[72] And every solve states its own bound: the certificate's strong-convexity modulus σ gives the **honesty radius** $2L_\Phi W/\sigma$ - the farthest a citizen's own weight can move the plan, scaled by their voice. A citizen's weight tilts a **nonlinear** term (the cap's bounded curve), not the decision itself, and L_Φ - the term map's own Lipschitz constant - is exact rather than assumed: the shipped cap shape delivers an owned want through a zero-labour column of coefficient exactly one, so $L_\Phi = 1/c$ at the spec's own catalogue scale c - one at the unstated default, corrected precisely wherever governance has set the scale elsewhere. Beside it the solve states its own honest floor: the certificate's duality gap, read once more through the same σ, bounds how far this **particular**, solved-to-tolerance plan already sits from the exact optimum.[73]

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[71] `cap`, code 5 in `module/playnet/planner/formulas/tension-kinds.scm`; the per-citizen emission and the wall in `engine.scm` (a want rides the citizen's own delivered good, `spec~aid`, through a zero-labour delivery column); the reference scale `set-cap-scale!`. The certificate's `TensionKinds.lean` derives the kind's gradient and curvature columns as the true first and second derivatives of its value.

[72] `SolverCertificate/QuantityAsCap.lean` (strategyproofness and the cap force the min-with-a-level shape; the level is the whole design freedom; `softLevelRule_strategyproof` - the swept wall's floored weight costs nothing, for **any** slack function, report-dependent or not), `CrossCell.lean` (`cross_cell_truth_dominant`), `MultiSupply.lean` (`shading_pays` - physical-unit truth-dominance fails under heterogeneous conversion - and `multi_supply_value_dominant`, the value-dominance that survives, at the technology's own fixed pulls). Sprumont 1991 characterises the anonymous member; Barberà-Jackson-Neme 1997 the full family - guaranteed minimums and ratified priorities included - and Arribillaga-Massó-Neme 2023 prove every member obviously strategy-proof. The one rule the family excludes is priority bought by the stated quantity itself, which is the manipulability.

[73] `need-displacement` answers `honesty` off the cached certificate - `sigma`, `l-phi`, `radius-whole-hand`, `solve-tolerance` - read live off every solve since `%verdict-fields` was found to drop `sigma/residual` from the cached plan entirely: the radius had been correctly PROVED but silently dark on every real solve, the pinned unit test having exercised only a stubbed certificate. The ⚖ chip now shows both figures. `SolverCertificate/InTheLarge.lean`'s `argmin_lipschitz_weighted_term` proves the general bound with L_Φ honest rather than assumed to be one (`argmin_lipschitz` alone takes the term map to be the identity); `dist_from_optimum_of_curvature_gap` transports the certificate's value-space gap $\|r\|^2/(2\sigma)$ (`DistCurvature.lean`) into the SAME argument-space currency, $\|r\|/\sigma$ - one modulus now sizes the honesty radius, the ask

The producer's hand is the second bounded hand, and the same argument holds it. A stance on a part is a signed portion of a hand held hour by hour (*The voice field*): over the parts any one validated hour falls in, portions sum to at most one, so a citizen who states more stances spreads the hand and one who states one louder quiets the rest, and scaling every stance changes nothing - the emphasis simplex, signed, with Jackson and Sonnenschein's linking argument carrying over unchanged. What the hand can move is bounded twice more. Its reach into the objective is τ_v: no coefficient leaves $[(1-\tau_v),(1+\tau_v)]$ of its clock hours, so what voice displaces - the standing plan against the same program at zero field - is at most τ_v times the labour of the two plans together, and a part's marginal displacement, $-\tau_v \sum_j \ell_j x_j$ over the columns it covers, is exact by the envelope and reads zero once the column's stance has saturated. And its reach into the **wants** is zero: the centring takes every statement about goods out of the push before the solve, so a producer's hand can lean a good toward one of its ways and can move neither how much of it is made nor what it costs at the worked mix - that stays with the consumer hand, where the theorems above hold it. A private preference over one's own slots is on neither hand; it is equal and bounded on the who rail, and it touches only its speaker's assignment.[74]

The strongest words stay deliberative. `held` and `floor` are the scope's statements: a citizen declares them only over a ratified grant that names its ratifier, and their quantities are facts - the prescription names the dose - audited against the observed record at Becker's rate, with the consequence factor the scope's own policy. One pattern stays visible rather than weighed: a citizen's own good delivered persistently above what they take - the conduit, a pad-and-gift channel - is recorded for the table's conversation.[75]

Sacrifice under scarcity is therefore a chosen pattern, stated plainly: between the scope's shared dimensions the assortment scales together; to the citizens within one dimension, service rises to each one's level, so a shortage lands on the largest asks first, tilted by voice. The strategyproof family spans many such patterns - guaranteed minimums, ratified priorities - and a scope ratifies its member the way it ratifies a kind. Proportionality **to the ask** is the one pattern outside the family: a rule that serves bigger statements bigger shares pays whoever states big, and a table that wants it buys it with audit labour rather than with shape.

The supply side is honest for the opposite reason: nothing connects it to a private return. The channel a market supplier manipulates - withhold, the price rises, and the rise is the supplier's income - has no analogue for the **means** a citizen holds: a dual is a planning signal and never an income, and custody of a contributed tool or space never vests. **Labour** is the one supply declaration that does carry a mechanical return - working raises producer voice, and (through the contribution share of drawable credit) the labour component of consumer voice too - but that return already tracks witnessed conduct exactly, since voice reads hours **worked**, never hours **offered**: there is nothing to overstate, only something to actually do. Supply declarations otherwise split into two kinds, and neither calls for mechanism design. An **offer** of availability is performative - it constitutes the fact rather than describing one, and free time is a want the objective already weighs - while stocks, skills, capacities and coefficients are **constative**, facts about the world, and those the witnessing rail audits: skills derive from the work log, stocks from the event-sourced stockbook the folds read, and a padded coefficient shows as low capture, and the plan leans on the recipe by what it delivers (§Character is a measurement). Mechanisms are designed for what cannot be verified; what can be is witnessed. One channel stays open regardless:

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wall's own softness (§ the swept-wall theorem above), and the certificate's optimality tolerance together. The `des` shape's squared-gap term has a different, unproven L_Φ; the correction applies only under the shipped cap default.

[74] `SolverCertificate/VoiceField.lean`: `share_sum_le_one`, `person_push_le`, `abs_push_le_one`, `warp_mem` (the coefficient's interval), `stance_lipschitz`, `what_statement_cancels`, `felt_equals_clock`; `Parts.lean`: `covers_meet`, `rung_covers_origin`, `covers_two_forces_release` (a union is unrepresentable; the meet of two parts refuses a clash). Run, with the hand's signed rescale, a fixed electorate's no-gain through many parts, rotation costing nothing and what voice displaces at the margin, as `tests/planner/test-property-voice-field.scm`.

[75] `ratify-kind` / `audit-kinds` on the planner service; the findings live beside the grants on the scope's plan-store. `docs/planning/identity/truth-telling.md` carries the design path and `quantity-as-a-cap.md` the reasoning, including why the natural consequence - the grant withdrawn - sets the audit rate at one.

consumer voice's drawable balance falls with what is **claimed**, so a citizen can strategically under-claim to hold their weight up across solves - a dynamic-game question the static dominance results below do not model, named here rather than assumed away.

What remains on the **means** side is withholding - the free-rider question - and its arithmetic is a theorem. Contributing capacity enlarges the feasible set, so it weakly raises the **aggregate** optimum; anyone whose own payoff is a nonnegative share of that aggregate weakly gains from anyone's contribution - a narrower claim than "everyone gains outright": the aggregate rising is compatible with a particular person's own realized bundle falling, if their stake in the outcome is not simply a share of the aggregate the theorem covers. For a holder whose payoff is such a stake plus the private use-value of the thing as they hold it, contributing is **weakly dominant** - against every profile of what anyone else does - PROVIDED two things hold, neither automatic: handing over custody forgoes nothing of that use-value, and accepting the thing into the pool attaches no obligation heavy enough to shrink what the plan could otherwise do (a means that arrives owing more maintenance than it is worth can make contribution strictly worse, and then the theorem correctly declines to fire). Both are design commitments the temporary- socialization ceremony has to keep, not consequences of "more capacity exists." Given they hold, withholding strictly pays on one exact condition: the private use-value forgone exceeds the holder's stake in the harmony gain. So the design's lever is that **price of generosity**, driven toward zero rather than compensated: custody stays revocable, the draw bounded, the exit free - the temporary-socialization credential makes lending an oven cost almost nothing. And the pressure that remains is social by design - the contribution component of consumer voice, the visibility of a quantifiable withholding - because a mechanism that **paid** for means-contribution would re-open the very private-return channel whose absence makes this side of the argument free.[76]

[76] `SolverCertificate/Contribution.lean`: `PlanEnv.welfare_mono` (a larger feasible set weakly raises its optimum), `no_stakeholder_harmed` (a stake in the aggregate is a Pareto move on that aggregate - `aggregate_gain_can_hide_individual_loss` is the witness that this is narrower than a claim about any one person's own realized bundle), `contribute_weakly_dominant` (dominance against every profile of the others - the holder needs no model of anyone else, provided both `henl` and `vKeep ≤ vGive` hold; `obligation_can_shrink_feasibility` witnesses that `henl` is a real hypothesis, not a free consequence of capacity existing), and `freeride_iff` - withholding pays iff $v_{\text{keep}} - v_{\text{give}} > \alpha \cdot \left(W_{\text{give}} - W_{\text{keep}}\right)$, the price of generosity against the holder's share of the harmony gain. `enlarge_of_ceiling` grounds the enlargement hypothesis in the wall shape the engine carries: raising a capacity ceiling **alone** is a superset of feasible plans - an added obligation is a separate, independently verified fact. The custody credential that drives $v_{\text{keep}} - v_{\text{give}}$ toward zero is `docs/planning/socialization/temporary-socialization.md`.

11. Appendix: Property, law, and borders

How the commons meets the law and property it has not yet changed, and how it handles its edges — borders, migration, and the topology of federation.

One object carries the whole appendix: the crossing. A scope boundary, the money membrane, a legal order and a national border are four edges of one federation, and something traverses each - a good, a sum of money, an activity, a person. At every edge the same three things are read. The **crossing** is the fact of the traversal, read off records the federation already keeps. The **demand** is what the far side asks of it - a flow to balance, a levy, a credential, a disclosure. The **pull** is what the traversal weighs in the plan, in labour-hours, beside every other pull. Goods at a scope boundary are the body of the document (§If it solves globally, why federate?); this appendix reads the other three edges with the same instrument - the property and money an organisation brings across, the legal order, and the border.

11.1. Interfacing with existing property

The social plan can coordinate production across any property form. A bakery, a clinic, a farm whose participants begin to form social plans with the means around them is, from that moment, a living scope in the federation. The legal title on the property is irrelevant to the operation: the property is **functionally social** because its use is determined by the plan, its output enters the total social product, and its workers carry producer voice.

A cooperative, a family workshop, a municipal hospital, an individually-owned tool shop - each can participate. The owner works, earns voice and credit like any citizen. What gets produced, how much, for whom, and who does the work is shaped by plan acceptance and producer voice. The private title remains; the content of ownership is social. Over time, as the plan coordinates more of the productive activity, the title becomes vestigial - the shell remains, the substance is the association.

What existing organisations bring into the federation is concrete: means of production, means of consumption, legal rights, recipes and knowledge, people and their offered availability, ecological agents, existing relationships, data, spatial catchment, cultural resources, financial assets, and existing obligations.[77] Each enters the plan as a resource with a specification; the existing world's artifacts become VF resources, custodied by the scope and administered like any other.

The social plan simply organises production better - matching hours to desires, allocating labour where it is needed, distributing the product through consumer voice. When the common plan works better than

[77] In detail: Means of production - tools, machinery, land, buildings, vehicles, raw materials, inventory, servers, infrastructure. Means of consumption - finished goods, food stocks, medical supplies, housing. **Legal rights as economic resources** - each modelled as a resource specification in VF: patents and licences (the right to produce), permits and zoning (the right to operate at a location), water rights, mineral rights, spectrum allocations, leases and contracts (access to means held by others), professional certifications (skill attestations from the existing system), domain names, trademarks, copyrights. Recipes and knowledge - production processes, workflows, technical documentation, software, research data, training materials. **People** - the offered availability of every participant, their skills, capacities, and time, the living labour that makes everything else productive. **Ecological agents** - when a farm joins, it brings the soil ecosystem, the water system, the pollinator populations, the seed bank, each an ecological agent under the scope's custody with its own regenerative envelope. **Existing relationships** - supply chains, partnerships, standing agreements with other organisations, becoming VF resource flows between scopes. **Data and records** - historical production data, demand patterns, environmental monitoring, accounting records, feeding the planner's understanding of $\mathrm{SNLT}_{\text{crystallized}}$, demand, and capacity. **Spatial catchment** - the physical location itself: a clinic at a specific intersection, a farm on specific soil, a workshop in a specific neighbourhood. **Cultural and artistic resources** - a theatre brings its repertoire and stage, a library its collection, a school its curriculum. Financial assets (cash, credit lines, investments) enter as external liquidity at their $\mathrm{SNLT}_{\text{crystallized}}$ of acquisition — through any of the three doors inward: acquisition by the federation's own labour, gift (a socialized purse), or restitution of money a member fronts ("Credits and external liquidity"). **Existing obligations** - contracts, service agreements, debts, modelled as VF commitments that the scope metabolises as it transitions.

the private plan, people follow it. The means of production are socialised in practice, by the free choice of the producers who use them.

The federation interfaces with the legal system through **legal adapters** - lightweight entities whose internal governance is the social plan. The legal form is an adapter; the social content is the commons.[78] Meeting regulatory obligations is just administration: the specification says what the regulation requires; the scope verifies compliance like any other specification.

A joining organisation arrives mid-crossing. A bakery with wage staff, a till and a lease trips the wage-labour, public-sale, state-money and property crossings on the day it joins, and its demand bill (§Law as derived specification) reads accordingly: payroll and its insurances, a seller's permit, a tax return, a lease in an entity's name. The federation carries that bill in full. Each wage contract is an existing obligation, a VF commitment honoured to its term; each sale to the public is the acquisition door of the money membrane (§Three doors inward, one door out), earning the money that pays the staff and the landlord. Transition is the same fold read over time: as a wage-worker becomes a member and draws credit, as a loaf moves from the till to the individually-claimable pool, the activity stops tripping its crossing and the bill shortens by that line. A scope in transition therefore has two readings at every moment - the share of its activity the plan determines, and the crossings that still bind the rest - and both are derived from the same records.

11.2. Property and law

11.2.1. Law as derived specification

A legal requirement is a property not of a recipe but of a **(crossing, jurisdiction)** pair, and this is what lets the federation meet every legal order through one seam. A recipe **trips crossings** - placeless facts about an activity, that it sells to the public, performs a regulated act on a person, draws on the shared environment - read off the pattern its own records show: the VF action, the kind of thing its specification says it acts on, whether an exchange brings something back, and whether custody and rights sit together. What the records do not carry - where a thing was cooked, whose it was, to whom it goes - a steward declares on the recipe. A jurisdiction answers each crossing with a demand. The requirement for a given activity in a given place is then never stored: it is **derived**, a fold, exactly as credit and every quantity on the surface is derived from the event log.

Each law is a **guarded demand**: a predicate over the activity's context - its place (the contained-in path), its agents (citizenship, membership), its assets - yielding a **demand** in one of four kinds. The state may ask the activity to **obtain** a credential (a competence licence, a permit, a registration), to **pay** a levy (a tax or duty), to **tell** a disclosure (a filing or report), or to be **forbidden** outright. Jurisdiction is one guard among several, so spatial law, personal law, and extraterritorial reach are one kind of object, and a **crossing** is a coarse index rather than a fundamental type. The whole legal exterior is a stewarded set of guarded demands, and the activity's bill is one fold over them:

$$\underbrace{\mathrm{bill}(a)}_{\text{the demand bill}} = \bigcup \{\mathrm{demand}_r : r \in \mathrm{rules}, \mathrm{guard}_r(\mathrm{ctx}(a))\}$$

[78] In practice, these adapters are cooperatives, associations, LLCs, or foundations registered cheaply in whatever jurisdiction is convenient. An adapter holds whatever interface the state requires: a bank account, a tax identity, a signatory for contracts. If an adapter is captured, regulated into uselessness, or becomes a liability, the federation dissolves it and registers another. The assets that matter - knowledge, recipes, specifications, the software, the social relationships - are already distributed across the federation's infrastructure. Some interfaces are expensive to re-obtain (regulatory licences, land title, banking relationships, professional accreditations) and justify a more durable adapter; everything else rotates freely. The apex form is **ownerless** - a non-charitable purpose trust whose beneficiary is a purpose, a memberless nonprofit, a self-owning foundation - and an operating entity (the one that signs leases and holds the bank mandate) is held by it, so the chain of ownership ends in a purpose, kept for it by custodian-stewards (§Standing: held by key, a mask at the door).

Three of the ways an activity meets the state are **chosen and activity-triggered**, so they fold: an **obtain** is a specification the scope verifies - a person-borne competence credential checked by the conformance closure that gates a skill, or an entity-borne permit held by the adapter; a **pay** is a levy and a **tell** a disclosure, answered by projecting the fields it names; a **forbid** gates the activity as hard as an ecological cap, a possibility-tension no desire can buy across. Two further modes stand **outside** the fold: **standing** - the capacity to be a suable party - is a fixture, maintained once per door (§Standing, below); and **subjection** - the state acting on you unchosen, by criminal law or eminent domain - is not derivable at all, met structurally rather than folded. Legal heterogeneity is otherwise internalised uniformly - the federation reasons about Alameda's food code and Brussels' data law with the same fold, the same administration that verifies any other specification.

What the seam furnishes is an **interface**, not legality itself. The registry holds only what stewards encode - sourced, dated, revisable, a cache of human legal judgment and never an oracle - and an unrecognised trigger fails safe to the **cannot-dodge** default rather than passing silently. So the federation meets each legal order at one door of uniform shape, while what stands behind that door stays local: whatever the people who know that order have written down. A universal interface to the law; its content always theirs.[79]

11.2.2. The catalogue of crossings

The crossings form one open catalogue, and every table in this appendix is a view of it. Each entry carries a **kind**, which says how the federation's own design stands toward the crossing. An **avoidable** crossing is one the internal metabolism leaves untripped: members draw on credit, associate as producers, and hold value by key, so a sale, a wage and a conversion occur at the membrane and nowhere inside it. A **by-choice** crossing follows an attribute the federation may take or leave - a title, a road. A **mirrored** crossing restates a competence or a safety the federation specifies for itself, and it is the kind an unrecognised trigger receives.

The kind is the federation's side of the reading. The other side is the jurisdiction's, and it lives in the registry as a rule with a source, a date and a confidence. One legal order reads a member's draw on credit as a household feeding itself, another as a sale for consideration; one reads matched hours as association, another as employment, and the credit drawn on them as income. Where a jurisdiction reads an internal activity as a crossing, its stewards encode that reading, the activity trips the crossing there, and the bill carries the levy, the permit or the return. **Avoidable** names what the design makes possible; what holds in a given place is the registry's answer for that place.[80]

[79] A legal requirement is a property of a **(crossing, jurisdiction)** pair, never of a recipe, so law need not multiply across recipes. The crossing vocabulary is open - an unknown trigger fails safe to **mirrored** - and rules are guarded demands in a `tension-registry`-style governance actor, event-sourced for a replayable legal posture; the demand bill and compliance alike are folds. The four demand kinds (**obtain**, **pay**, **tell**, **forbid**) are the chosen, activity-triggered modes of state interaction; **standing** and **subjection** are the two that stand outside. See `docs/legal/legal-crossings.md` and `module/playnet/planner/{crossings,legal-registry,reform}.scm`.

[80] The catalogue is `*default-crossings*` in `module/playnet/planner/legal/crossings.scm` - open, extended per jurisdiction through `register-crossing!`, an unknown crossing reading as `mirrored`. The readings that vary most by place: some food codes treat any exchange for consideration - and credit is consideration - as a sale; tax authorities commonly treat scrip and barter as income; employment status commonly follows the economic reality of the work, whatever the parties call it; and health authority reaches high-risk foods (raw milk, meat) and any operation where someone falls ill, sale or no sale. What trips a crossing is one table of patterns in the same file (`*crossing-patterns*`), read over a process's flows, a resource at the cell, or a record a door writes; a `crossing:` tag declares what the records do not carry, and `make lint-recipes` says which is which for every recipe. One part remains to be built: the handover - who receives a good and what comes back - which the recipes do not yet state, so `public-sale` is read from a recipe's exchange and is not refined by membership (`docs/legal/legal-crossings.md` §2).

a crossing — what trips a wrapper	the wrapper it pulls	its kind — how the design stands toward it
sell to a non-member, or for state money	sales licence · entity	**avoidable** — members draw on credit - a sale is the membrane's acquisition door
sell or serve food to the public	establishment permit · food-handler card	**avoidable** — members eat from the individually-claimable pool
convert to or from state money	entity · KYC per event	**avoidable** — value is held by key - a draw of external liquidity is the one conversion
employ non-member wage labour	payroll · compensation insurance · workplace code	**avoidable** — producers associate and draw credit
own or lease real property · contract a legal-only outsider	entity - lessee or title-holder	**by-choice** — a lease passes the counterparty door, a title opens the registry
a titled vehicle on public ways	registration · insurance · driver licence	**by-choice** — by attribute - foot and bicycle trip nothing
press a program, organise or strike collectively	a union's or an association's free voice · a charity's limited one	**by-choice** — voice is every citizen's, spoken from the plan
share a lent thing's running cost, a fixed sum by period	cost recovery · a nominal sum where consideration is asked	**by-choice** — the commune bears the cost of what it uses; the owner keeps the right and takes no rent
pay for a lent thing by the use made of it	a lease · the lessor's tax and the tenant's protections	**by-choice** — a lend carries no price per use - what tracks the use is rent
run a service that people in other places use, and answer for what it holds of them	a controller's and a host's duties · a representative where none is established	**by-choice** — the operator is a body the law addresses like any other: its duties are read for each place it serves, and its standing is the stance of that reading
build or occupy an inspected structure	building · fire · occupancy	**mirrored** — safe to occupy - the scope's own specification
a regulated act **on a person**	practitioner licence	**mirrored** — the act is skill-gated
a dangerous or controlled thing	arms · drugs · slaughter	**mirrored** — its safety specification
draw on or emit into the shared environment	environmental · water · air	**mirrored** — a possibility-tension
process other people's personal data	data-protection duties	**mirrored** — a read is a capability, scoped to its purpose
insure or pool risk for others	insurance licence · solvency rules · supervision	**mirrored** — the reserve is held in stock by the plan - no premium, no promise to pay

Figure 25: **One catalogue, read by crossing.** Every branch runs **internal** in its core - held by key and by plan, member to member - and pulls a wrapper only where it trips one of these crossings, and only for that crossing's object. Four are **avoidable** (green): they occur at the membrane, where the federation sells outward, converts, or carries a joining firm's payroll, and wherever a jurisdiction reads the internal activity as one of them. Two are **by-choice** (grey), following a title or a road. Five are **mirrored** (blue) - the built structure, the regulated act, the dangerous thing, the ecological draw, another person's data - each a competence or a safety the federation also specifies for itself, whose legal and internal specifications §Dual power sets side by side.

Runs internal · trips no crossing by design

- **knowledge & digital** — software, writing, research, design, recipes
- **governance** — planning, voice, credit, matching, mediation
- **adult education** — apprenticeship, skill-sharing, mentoring
- **food for members** — grow, cook, preserve, distribute on credit
- **farming & land care** — crops, soil, seed-saving, forestry
- **craft & making** — textiles, furniture, tools, pottery, soap
- **repair** — of the commons' own unregulated goods
- **care & mutual aid** — elders, companionship, small-ratio childminding
- **arts & culture** — performance, library, archive, in member spaces
- **internal logistics** — VF transfers, warehousing, foot & bike
- **recreation & hospitality** — communal meals, play, gatherings
- **ecological stewardship** — composting, watershed & soil care
- **self-use energy** — rooftop solar, micro-generation
- **treasury** — credit and crypto, key-held

Pulls a wrapper · at one crossing

avoidable

- **public sale / market** — sales licence + entity
- **a public table** — establishment permit, inspection, the carded hand
- **the fiat on-ramp** — one adapter facing the bank, the person who crosses authenticated
- **a joining firm's payroll** — wage contracts honoured to their term

by-choice

- **premises & land** — lease or title, utilities, insurance + entity
- **road transport** — vehicle registration, insurance, driver licence
- **a union, an advocacy body** — registration, its own books, apart from the charities
- **utilities on a lent room, upkeep of a lent van** — flat, never by the use
- **money claimed per use of a granted thing** — a lease in all but name
- **a gateway offered across a border** — a representative, points of contact, a notice of breach

mirrored

- **construction** — building/fire/occupancy + engineer's stamp
- **regulated trades** — electrical, plumbing, gas-fitting
- **health & medicine** — practitioner licence, the named clinician
- **dangerous goods** — arms, pharmaceuticals, explosives, slaughter
- **extraction & emission** — mining, large energy, discharge, fishing
- **records about non-members** — lawful basis, retention, the subject's access
- **a mutual, an insurer** — licence, reserves, supervision

Figure 26: **The same catalogue, read by branch.** The left column is the federation's internal metabolism: by its own design it trips no crossing, holding by key and moving by plan, and a jurisdiction that reads one of these activities as a crossing adds its line to the bill there. The right column is the remainder, each branch wrapped at one crossing and grouped by that crossing's kind - **avoidable** (green) where the federation faces outward, **by-choice** (grey) where it takes a title or a road, **mirrored** (blue) where the licence restates a competence or safety the federation already imposes. Health and construction are wrapped at their core; every other branch is wrapped by an attribute - a sale, a road, a structure, a danger, an emission - and runs on the left without it.

Food shows the pattern whole. Food law fastens on the establishment that serves the public for commerce. A commons feeding its own members from the individually-claimable pool, drawn on credit, stands where a household or a private club feeding itself stands, and a legal order that reads it so summons no establishment licence. The food crossing fires when the federation faces outward - sells surplus to non-members, opens a public table, takes fiat at a counter - and in any jurisdiction whose code reads consideration of every kind as a sale. The permit, like the entity, is the cost of a door opened.

An **obtain** comes in two kinds, and a licence shows both. A **facility licence** - an establishment permit, a seller's permit, an occupancy or processing licence - rides the **entity**: the adapter that signed the lease holds it, the inspector reads the premises, and the public record names the establishment. A **competence licence** - a food-handler card, a practitioner's certificate - rides a **named person**: it is a skill attestation, the state's own rendering of the federation's skill-conformance credential, surfacing its holder as **trained**. The producer already carries the internal attestation; the card is the same competence spoken in the state's vocabulary.

11.2.3. Standing: held by key, a mask at the door

Standing is the fixture the fold leaves outside: the capacity to be a party the law can address. The commons holds almost everything without it. Inside, ownership is possession, capability, and the plan: a recipe is held by being distributed, a tool by being used, the treasury by holding its keys. A legal person is, in the word's first sense, a *persona* - the **mask** an association wears to speak at a door that opens only to one - and there are three such doors: a **counterparty** that contracts only with a suable party (a landlord, a licensed supplier, a bank), a **state registry** where ownership is a public record (real property, vehicles), and a **liability sink** - the entity that stands as defendant, so a tort claim meets a limited pool and an insurance policy. Crypto, recipes, knowledge, data, and untitled tools pass none of these doors and wear no mask; they live in the cryptographic and social layer, held by key and by plan.

An adapter is therefore **per door**: a thin entity holding what its door touches - a title, a contract, a tax identity, the money that crosses there. Two records sit behind it, each with its own readers. The **legal books** are the adapter's, kept in the state's vocabulary and open to whomever the law entitles. The **social ledger** - membership, matched hours, drawn credit, the whole of who did what and who received what - is the federation's own event log, held in the credential substrate. Where a door's bill carries a **tell**, the ledger answers it by **projection**: the filing renders exactly the fields the demand names, a field under the pseudonymity floor goes out under its stable pseudonym, and a field the form asks that the federation holds back surfaces as a **withheld** tension, shown in the filing's preview for governance to decide. Disclosure follows the read - the inspector reads the premises, the tax office the return, the bank the person who crosses - and each reads its own part.[81]

Each mask is ownerless and stewarded. Its form carries no beneficial owner - a non-charitable purpose trust, a self-owning foundation, a code-managed DAO-LLC - so the chain of ownership ends in a purpose. Its **control** is a key quorum: an M-of-N threshold of **custodian-stewards**, members who act as fiduciaries to the purpose, hold no ownership, take no benefit, and sign knowing what they sign for. The quorum is a safety property - the property moves only when the threshold agrees, and a steward rotates by governance while the entity stays whole. Where the law names a controller - a beneficial-ownership register, a bank mandate, a recorded deed - the stewards are who it names, and the role the register records is the role they hold. This is the recognised form of §Social property, recognized practised ahead of its statute: the same purpose, the same guardians, the same asset lock, assembled today from a trust and a company.

The legal surface is exactly as large as the doors the federation opens. Value held by key passes no bank. A leased workshop passes the counterparty door and leaves the registry shut. A published recipe has no registrant. Each door opened adds one adapter, one bill and one set of disclosures, and each passage of state money authenticates the person who crosses, once per event (§Three doors inward, one door out). Fewer doors is less to administer and less to disclose - a single optimisation - and at each door that stays open stands a steward the law can address.

[81] A filing template declares its own disclosure surface - which keys it discloses, which it floors - and `render-fields`, `template-disclose` and `minimize-disclosure` (`module/playnet/planner/legal/filings.scm`) derive the filing and its withheld fields from that one declaration; `mask-subject-of` (`adapters.scm`) routes each entity demand to the per-door adapter that covers it.

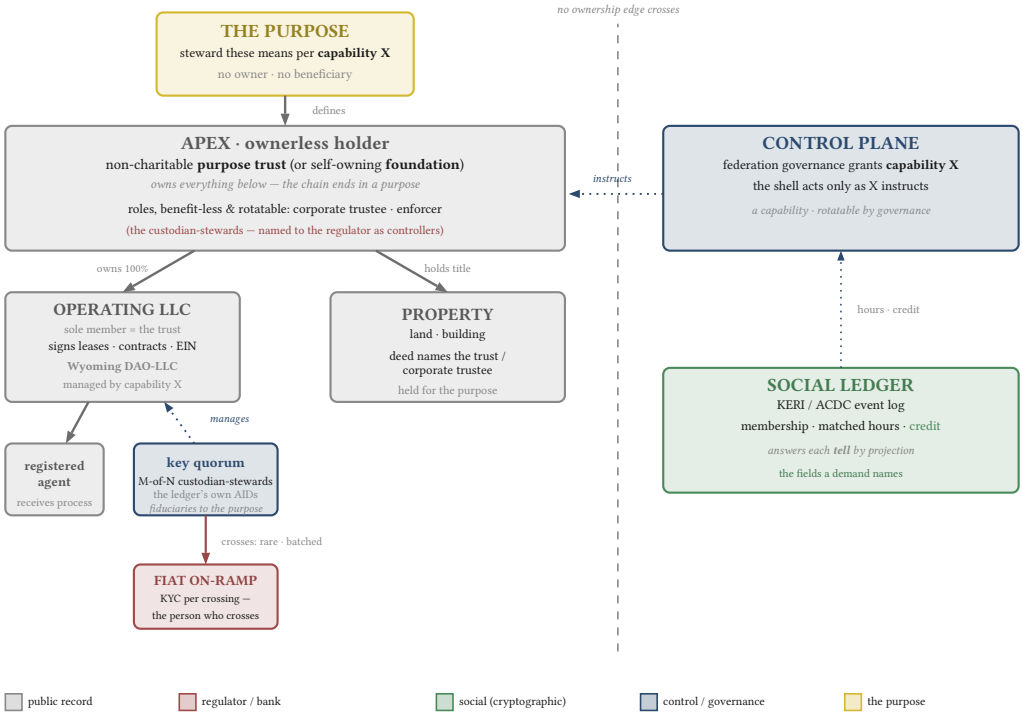

Figure 27: **The legal constellation.** The chain of **ownership** (grey, solid) runs from a purpose down through an **ownerless apex** - a non-charitable purpose trust or self-owning foundation - to the operating LLC and the property, and **ends in a purpose**. **Control** is a key quorum - an M-of-N threshold of custodian-stewards, the legal form a DAO-LLC managed by that quorum - fiduciaries to the purpose who hold no ownership and take no benefit. Three planes read the structure, each its own part. The **public record** (grey) shows the entities and the registered agent. The **regulator and bank** plane (red) reads the stewards as controllers, and at the **fiat on-ramp** authenticates the person who crosses, once per passage of state money. The **social ledger** (green) - KERI/ACDC, holding who worked and who drew credit - answers each **tell** by projection, the fields the demand names. The control plane (blue, dotted) couples the structure by **capability**: governance instructs it and reads the ledger, and ownership stays with the purpose. One operating mask is drawn here; in practice the federation wears a thin one **per door**, each holding only that door's object, while the treasury, the recipes and the untitled tools wear none.

A worker is protected through the same three structures. Against **third-party harm** - a customer poisoned, a passer-by hurt - the **liability sink** answers: the licensed entity is the defendant and its insurance pays, so a suit meets a pool and a policy. Against **exposure** - a card or a filing carrying more of a person than its purpose needs - **projection** answers: the food-handler card surfaces a carded, competent hand, the return carries the fields the return asks, and membership and credit rest in the ledger until a demand names them. Against **injury to the worker's own body** the commons' own care answers - communal provision, the cover an associated member holds - and where a jurisdiction reads the member as an employee, its compensation insurance joins the bill beside it. The three protections a wage-worker draws from an employer are assembled from the entity, the ledger and the commons.

11.2.4. Dual power: legal, internal, desired

A mirrored crossing has two parts. Part of a legal specification is safety the federation shares; **part** is duress it does not. Set three specs beside each binding law - the **legal** spec L (what the state demands), the **internal** spec I (what the scope already verifies of its own accord), and the **desired** spec D (what the federation wants the constraint to be) - and the difference between them is a **reform programme**, derived rather than authored.

region of the law	what it is	the move
$L \cap D \cap I$	wanted and absorbed	**redundant check** — demand recognition of our inspection, withdraw the state's; a state function withers
$(L \cap D) \setminus I$	wanted, not yet absorbed	**absorb first** — build the inspection; a precondition, not yet a demand
$L \setminus D$	unwanted (duress)	**transform law** — the duress-gap, what we do only for the law; advocate $L \to D$
$\emptyset$	nothing to press	**keep** — accepted, or an existence wrapper that certifies no spec

Figure 28: **Reform is the diff of three specs.** The **legal** spec L partitions, against the federation's **internal** spec I and **desired** spec D, into a redundant core to demand withdrawn, a roadmap to build, and a duress-gap to transform; a demand whose regions are all empty is **kept**. A single permit spans the first three - a safety-core it reproduces, an exclusionary shell it resists.

Where the federation **wants** a requirement and has **absorbed** it - reproduced it as its own specification and verifies it - the state's check is **redundant**, and the demand is its withdrawal: a state function withers. Where it wants the requirement but has not yet built the inspection, the region is an internal roadmap, the precondition for that demand. And where the law demands what the federation does **not** want - the building code that bars housing the homeless - the region is a **duress-gap**, what it does only because the law forces it, and the demand is that the law be transformed toward D.

Dual power is read as a move. The regions are sets of clauses, and clauses are not commensurable units over a common denominator, so no ratio of them is taken. What the diff yields is a **move**, by a plain reading of which regions are empty: **wither** when the federation provides all it wants of a law and nothing remains to build; **absorb** while something it wants is still to build; **transform** when only the shell is left, borne or rejected; **keep** when there is nothing to press. The internal spec I is itself derived: a clause is absorbed when a way the members actually run performs it, so the move follows practice and a place's program is whatever its ways have earned. A clause the vocabulary has not yet mapped to any aspect stands in a fifth state, **not yet mapped**, and is shown as such rather than folded into any move.

Permits are the sharpest case. A permit **is** a state inspection against a specification - the same act the federation's administration already performs, and performs **continuously** where the state inspects **periodically** - so its redundancy is the most demonstrable and its withering the most concrete, the state's inspectorate replaced by the federation's own audit layer. The diff bites on **spec-bearing** demands - permits and competence licences; a bare entity or registration certifies no spec, and stays the doorkeeper floor rather than a reform target. The same diff covers the **non-spec** demands too - a levy or a disclosure cannot be self-verified, so it has no redundant region, only **kept** (what we accept) or **transformed** (the duress-gap), which is where a forced disclosure or an extractive tax is named and contested. What the federation cannot derive is D itself: the diff and the readiness are computed, but the **wanting** is governed - priority and stewardship, never an oracle - and a duress-gap, once named, is legible to the state as well, so the federation presses it in the open: the claim is the published diff, and its standing is the inspection the federation already performs. The federation's growth in its own inspection is, precisely, its growing licence to let the state's redundant functions fall away.[82]

11.2.5. The socialization of property

To socialize a means of production is to move its **control** from private will to the social plan - its use plan-determined, its custody a scope's, its key a quorum's. That change lives in the **control layer**, and the law never sees it. **Social property**, here, is not a title but a relation: a thing whose use the plan

[82]The reform programme is the diff of three spec-sets per binding constraint: L (legal, from the registry), I (internal — what the scope verifies, read off the audit-intensity layer), D (desired — governance-declared). Its regions (`reform-regions`), the move (`reform-move`: wither / absorb / transform / keep) and the program sorted by move are folds in `module/playnet/planner/legal/reform.scm`; clause⟶aspect mapping and practice-derived I live in `knowledge/legal-aspects.scm` and `knowledge/activities.scm`; see also `docs/legal/legal-crossings.md` §8.

determines and whose custody a scope holds, whatever the deed still says - so the title goes vestigial, the shell remaining while the substance becomes the association.

The federation leaves private property as a legal form in place, and it **empties** it. The line it draws is the one the categories already draw: a means of consumption claimed on the individual rail is the member's **personal property** and stays so - drawing one's credit-share trips no crossing, for it is an internal claim and not a sale - while the means of production and the communal means of consumption become **social property**, plan-determined and scope-custodied. Private property as an institution persists, drained of the means of production; personal appropriation of consumption goods is left untouched.[83]

There is one seam at which socialization is **forced** to become a legal act: **titled durable property** - land, buildings, vehicles - where the state's registry recognises only a named owner, never the plan. To make a means of production social and **durable** - outliving the owner who brought it in - its title must be restructured so the owner is a **purpose, not a person**: the ownerless mask. Until then the social property is hostage to whoever's name is on the deed. This registry crossing is the **definitive moment** of legal socialization; everywhere else, socialization is a control-layer change the law never witnesses. Three transformations exhaust the field:

transformation	what	the act
acquire a mask	titled durable means of production and communal facilities	re-title into an ownerless form — the definitive moment
dissolve a form	private patents / IP; a joined firm's title	release it, or let the shell go vestigial
no legal act	untitled means of production, materials, services, skills, recipes, individual consumption	socialise in the control layer, by the plan alone

Figure 29: **Three transformations exhaust the socialization of property.** Only **titled durable** means of production and communal facilities force a legal act - the ownerless mask, the registry's definitive moment. Privatised intangibles socialise by **dissolution** of their form; everything else - the bulk of the metabolism - socialises in the control layer alone, by the plan. Individual means of consumption are never socialised: they remain personal property.

And the deepest reading closes the loop with dual power: **the ownerless mask is itself a duress-gap.** The federation holds land in a purpose trust **only** because the law forces a named owner; what it desires is the law's recognition of social property as a category of its own. In the reform diff the legal spec demands an owner, the desired spec is recognised commons-property, and the gap - the whole apparatus of trusts and key-quorums - is duress. So the deepest property reform is **transform-law**: the legal recognition of plan-custodied social property, after which the masks wither. The per-door masks are the **transitional** form the federation wears while the law still forces a name.

Socialization-in-use is immediate - the moment producers form a plan with the means around them, those means are functionally social; socialization-in-title is deferred and defensive, the ownerless mask acquired only when the named owner becomes a vulnerability. And as free time rises and dual power grows, the legal residue shrinks toward nothing - the title was always the shell, the plan always the content.

11.2.6. Social property, recognized

The reform that crowns this is a single recognition, and its good fortune is that almost every piece already exists in law somewhere - so it is **assembly, not invention**. Today a deed must name an owner, and the federation answers by nesting a purpose trust so the owner becomes a purpose. Recognition collapses the nest: a new title category - social property - in which the register records the **purpose and its custodian-stewards**, not an owner. The statute fixes four features:

[83] The federation's two rails render the distinction operational: the individual-claimable rail covers personal consumption, and social-property custody covers the means of production.

No beneficial owner. The entity is self-owned, purpose-owned; there is genuinely no natural person with an equity interest to disclose.

Asset lock. The property cannot be sold into private hands or distributed as profit - it is bound to the commons purpose in perpetuity.

Custodian-stewards, not owners. Named people act as **fiduciaries to the purpose** - guardians with power to contract, sue, and be sued on its behalf, who hold no ownership and take no benefit.

Plan governance recognized. The entity's constitution **is** the social plan - voice, custody, refederation - accepted as valid internal governance.

The single demand decomposes into a handful of statutory asks, and the elegance is that **each dissolves exactly one duress-gap the model named**:

the statutory ask	the duress-gap it dissolves
a **commons-title** registry class — deed reads "social property, custodian = scope"	the **own-property** mask nest
an **asset-locked, purpose-bound** entity (steward-ownership style)	the purpose-trust-over-DAO-LLC nesting
a **beneficial-ownership carve-out** — "no owner; custodian = role"	the **tell** — a register that asks for an owner
public-benefit, non-distributing tax treatment	the **pay** — property-tax extraction
custodian standing in adjudication	the **standing** mode — suable without an owner

Figure 30: **The crown is assembly, not invention.** Five statutory asks, each dissolving exactly one duress-gap the model named - the **own-property** mask, the **tell** disclosure, the **pay** levy, the **standing** fixture - and each already existing somewhere in law. The federation asks the legislature only to federate them into one social-property category.

And every fragment is already on the books - which is why this is a realistic **transform-law** target and not utopia: the ownerless purpose-binding of the Islamic *waqf*, the communal land title of the Mexican *ejido*, the statutory **asset lock** of the community land trust and the British community-interest company, the no-private-extraction of German *Verantwortungseigentum* (steward-ownership), the custodian-not-owner of legal personhood for rivers and ecosystems with human guardians, the recognised plan-governance of the Wyoming DAO LLC, and the beneficial-ownership and property-tax carve-outs already granted to purpose-bound nonprofits.[84] The reform asks the legislature only to **federate these existing forms** into one coherent category - a federation of fragments.

Once it exists, the transitional apparatus withers: direct commons title in place of the purpose-trust-over-DAO-LLC nesting, a register entry that reads **custodian** where it asked for an owner (the carve-out), the property-tax exemption, and custodian standing in place of a trustee standing in for an owner. The masks are the workaround, and recognition retires them. Two things remain, and we name them. **Custodians are named** - as they are today: the stewards of the mask already hold the guardian's role, fiduciaries to the purpose who **hold no ownership and take no benefit**, and recognition gives that role its own line in the register. And **it requires actual legislation** - the one demand in the whole reform programme the federation cannot absorb its way to, won only by political power and the moral clarity of the claim: *property that serves a community and provides its own welfare should be recognised as social, not forced to*

[84] Each fragment, already statutory or judicially recognised: the Islamic *waqf* - inalienable property dedicated to a purpose in perpetuity, ownerless, administered by a custodian (*mutawalli*); the Mexican *ejido* - communal land title held by a community, not individuals; the community land trust and the UK *Community Interest Company*, whose **asset lock** binds property to a community purpose by statute; German *Verantwortungseigentum* / steward-ownership, a drafted "company with tied assets" that cannot be sold and whose profits cannot be privately taken; legal personhood for rivers and ecosystems with human *guardians* (the Whanganui in Aotearoa New Zealand, and rulings in Ecuador and India); the Wyoming DAO LLC, recognising plan / algorithmic governance as a valid constitution; and charitable / nonprofit beneficial-ownership and property-tax exemptions.

masquerade as someone's private estate. It is the lone reform of the **transform-law** quadrant that no self-verification can reach - it has to be won.

So the crown is concrete: a social-property title class, an asset-locked purpose-bound entity with custodian-guardians, a beneficial-ownership carve-out, public-benefit tax treatment, and recognised plan governance - each already existing in fragments, assembled into one legal category that lets the commons hold property **as social property, without wearing a mask**.

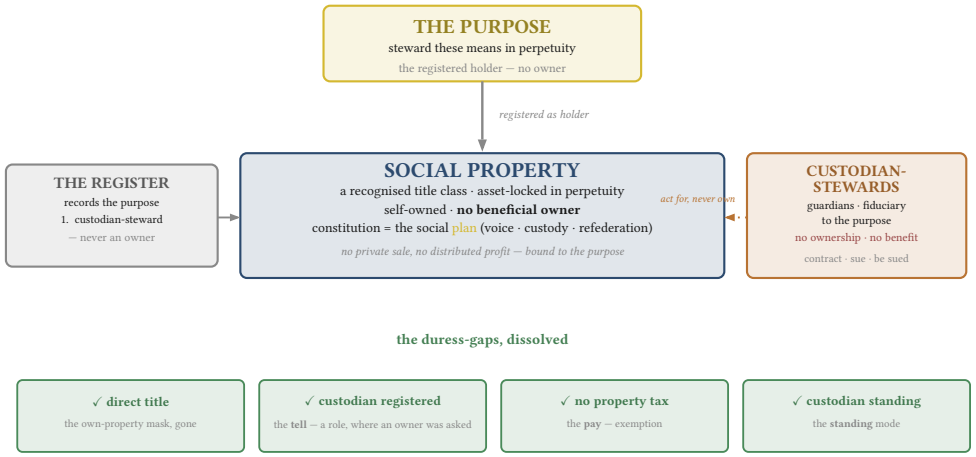

Figure 31: **The recognized social contract.** Under a **social-property** title class the register records the purpose and its custodian-stewards - guardians who act for the purpose and never own it - holding the property asset-locked in perpetuity with no beneficial owner and the social plan as its constitution. The four duress-gaps the mask stands in for - the owner the register asks for, property tax, trustee standing, the indirect title - dissolve at once. Assembled from forms already on the books (*waqf*, *ejido*, CLT / CIC, steward-ownership, river-guardianship, the DAO-LLC), it lets the commons hold property **as social property, without a mask**.

11.3. The border is a crossing

A country sits where three of the federation's structures try to coincide and never quite do. It is a **place** - a node in the contained-in tree, a region of the spacetime lattice. It is a candidate **scope root** - a cluster of activities with its own custody and governance. And it is a **jurisdiction** - the index on which the legal exterior keys its guarded demands (§Law as derived specification). The three are **distinct partitions of the same world** - geography, doing, and law cut it three different ways - and the questions a border raises live exactly where the three diverge. The federation already holds the machinery; the border is where two pieces of it compose.

A national border is a crossing on a spatial arc. Transport is an ordinary activity - a column of the technology matrix that consumes a good here and produces it there (§Everything is an activity) - and a legal requirement is a property of a (crossing, jurisdiction) pair, never of a recipe. So an arc whose two endpoints lie in different jurisdictions **trips a border crossing**, and the crossing answers with a demand in one of the four kinds the seam already knows: a tariff is a **pay** (a levy riding the arc), a customs or phytosanitary certificate an **obtain** (a specification the scope must satisfy before the good may move), an embargo a **forbid** - a possibility-tension as hard as an ecological cap, that no desire can buy across. The plan then **routes around an expensive border exactly as it routes around a congested road**: the measured arc already prices each relocation by its physics - trip-time, distance (§The measure of the arc) - and the border adds a legal coefficient to that same arc. A tariff, read this way, is **a dual on a crossing** - a labour-hour rent levied at a line on the map, kin to the **ecological rent** a strained envelope adds to a

good (§Concrete and abstract labour).[85]

Building on a border falls out of the same fold. An activity's bill is the union of every guarded demand whose guard its context satisfies, and the guard most relevant here is **place ∈ jurisdiction-path**. A facility whose cell straddles two countries satisfies **two** such paths at once, so its bill is the union over both rule-sets - doubly-wrapped, by exactly the mechanism that wraps a single-jurisdiction activity once. The border-straddling cell is simply a context that trips more guards.

11.3.1. The migrant is a rider

Material is local; labour is universal - the UC pools every citizen's offered availability as one fund and deploys it wherever the work-intents fit. The border is where **labour is universal** meets its hardest test, because a person crossing it is at once the **strictest perishable good on the lattice** - an unspent hour is simply gone, it cannot store forward - and the bearer of the **heaviest crossing** the legal exterior imposes. The machinery is already built: a worker is a rider (§The worker is a rider), a presence relocated on the same arcs that carry cargo, supplying labour where it rests. Put a jurisdiction levy on the worker-arc and **migration** is the rider tripping a crossing - and the plan weighs, in one optimisation, **carry the worker across the border** against **carry the goods to where the worker already is** against **train a second worker locally** - the doctor-far-from-the-clinic trade-off, now with a border-levy on the commuting arc.

This is where **international solidarity becomes a setting rather than a sentiment**. The federation chooses, crossing by crossing, which border-arcs it treats as **internal** - levy zero, the worker-arc priced by its physics alone - and which stay at the blanket's edge. To lower a worker-crossing's levy toward zero is to **pull that border inside the federation**: free association of producers across it, the same egalitarian matching that already ignores scope boundaries now ignoring the national one too.[86]

11.3.2. The metabolic border

The sharpest question the border raises is structural, and the federation already carries the measure for it. **Metabolic coherence** $M(s)$ is the ratio of a scope's internal flows to its total flows, and refederation auto-proposes a split or merge wherever it would raise that ratio (§Metabolic coherence, §Refederation). A political border drawn across a **living economic loop**

- a shared watershed, a cross-border industrial corridor, a valley whose

field and mill and table sit on different sides of one line - **severs flows that belong together**, and every severed flow is one that crosses the scope boundary rather than closing inside it. So a scope drawn on the **border** reads as **low** M; the same activities re-cut around the **loop** read as **high** M. The border shows up in the model as **coherence loss**, and the federation's own arithmetic argues for a scope drawn around the real metabolism rather than the inherited jurisdiction - the closed loop that collapses the division between those who plan it and those who work it does not stop at customs.

[85] An international border is one more crossing-attribute, and the jurisdiction guard tests which side of the line each of an arc's endpoints sits on - so spatial law and the border fold into the same `(crossing, jurisdiction)` registry as a sales licence or an ecological draw. See `docs/legal/legal-crossings.md` and `module/playnet/planner/{crossings,legal-registry}.scm`.

[86] The world market, in the *Grundrisse*, is "directly given in the concept of capital itself"; the federation inverts the figure - free association across borders given directly in the concept of the commons, the producers universal where capital made only its market so.

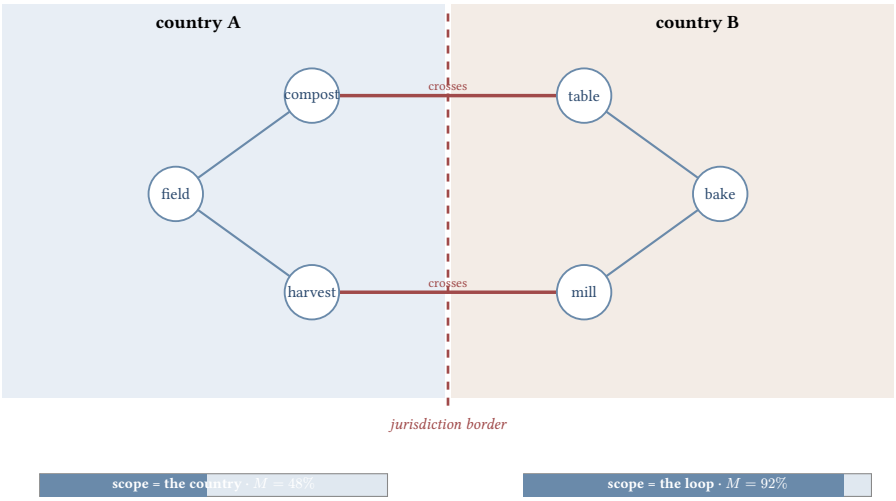

Figure 32: **The metabolic border.** A closed food-and-nutrient loop - field, harvest, mill, bake, table, compost - straddles a jurisdiction border (red, dashed); two of its flows cross the line. Cut the scope on the **border** (left bar) and those crossings become boundary-crossing flows, so metabolic coherence $M(s) = F_{\text{internal}}/F_{\text{total}}$ falls; cut it on the **loop** (right bar) and the cycle closes inside one scope, M climbs. Refederation reads the live flows and proposes the higher-M cut, so the federation's own coherence measure argues for a scope drawn around the metabolism, not the map. The jurisdiction stays put: a loop-scope re-cut across the line still pays a border crossing on every internal flow that traverses it - and that gap, between the metabolic boundary and the political one, is a **duress-gap**.

What refederation **cannot** do is move the jurisdiction: the legal line stays where the law put it, and a scope re-cut across it carries a border-crossing on every internal flow that traverses the line - the coherence the metabolism wants, taxed by the law that does not. That gap, between the metabolic boundary and the political one, is precisely a **duress-gap** (§Dual power), and naming it is the first move of the reform below.

A transboundary commons is one pooled budget. A river, an airshed, a fishery, the atmosphere itself - drawn on by scopes in several countries at once - is exactly the **distributed-draw** shape the federation already handles (§Shared budgets): one shared stock, one gap, one possibility-tension that out-pulls desire before the envelope breaks, **regardless of how many jurisdictions hold the straws**. The possibility-tension does not count borders. The atmosphere is the universal case - one envelope, every federation on Earth drawing against it - and the only thing the border adds is a **governance** question, never a mathematical one: who holds stewardship custody of an agent no single jurisdiction contains. The architecture answers in its own terms - a stewardship scope whose contained-in parent is the shared agent itself rather than any country, custody following the metabolism rather than the map.[87]

11.3.3. No core, no periphery

A legal levy rides the border arc; **unequal exchange does not**. The transfer of value the world market works across its frontiers - equal labour exchanging at unequal prices, the periphery's hours draining toward the core - is a category of the **commodity form**: it presupposes exchange-value, a price, a wage, and the equalisation of profit across the line. Inside the commons none of these exist. A good costs its crystallized SNLT, the **same number whoever draws it**; credit is one-directional and redeemable at that cost everywhere in the acceptance domain; and a grain flow from one country-scope to another is a

[87] Marx, *Capital* Vol. III, Ch. 48, on the associated producers "rationally regulating their interchange with Nature" (cited above): the transboundary commons - the Mekong, the Rhine, the global fishery, the atmosphere - is the distributed-draw failure mode at planetary scale, and the **pooled budget** in stewardship custody is the same instrument that handles a single watershed, the envelope indifferent to how many jurisdictions draw on it.

VF **transfer** settled by the one ledger, where **no one pays anyone** (§Production, distribution, exchange, consumption). There is no exchange for the inequality to inhabit, so **there is no core and no periphery between members of one federation**. Socialising production and distribution does not make value-drain visible - it removes the ground it stood on, exactly as a transparent plan dissolves the circulation sphere and its **faux frais** (§Transport and storage are one recipe).

What the border arc **does** still carry are two real costs, neither a transfer between the two sides. The first is the **legal levy** a jurisdiction extracts - a tariff, a customs charge, a dual on the crossing, but **the state's** extraction, not the periphery's drain. The second is the honest **transport labour** of distance: a remote region's **delivered** good carries more crystallized SNLT because carrying it there is real social work - geography, counted once like any recipe's, never false - and its citizens draw it at that true delivered cost, or the plan weights remote provision up in desire and the federation chooses to bear the carriage.

What **can** genuinely persist across a border is not unequal exchange but **uneven development of the productive forces** - one region thick with accumulated means of production, trained skills, and worked soil, another thin in all three. This is a real material asymmetry, and the federation answers it with the instruments it already holds rather than by auditing a transfer. **Expansion** (§The categories of social reproduction) builds means of production and forms skills where they are scarce, pulled by desire; and social desire satisfaction Φ, the harmonic mean that collapses toward whoever is worst off, together with the solidarity and citizenship components of consumer voice, steer the social product **toward** the thin region rather than away from it. The international task is therefore to **equalise development** - raise the floor where the productive forces are weak.[88]

11.3.4. One commune, or many federations

The document draws a single **Universal Commune** at the root and places **other federations** outside the Markov blanket, beside nature and markets (companion *planner-as-active-inference*). The border is the choice of **where to draw that blanket** - and it is a choice. To keep a neighbouring federation **external** is to coordinate with it as the commons coordinates with any commodity economy: a **sink** that admits resources inward at their crystallized SNLT of acquisition, the boundary sovereign (§Credits and external liquidity). To pull it **inside** is to make the two one contained-in tree under one root - the global UC the single-program decomposition already assumes (§The whole plan in one solve), each country a scope solving its own corner, coupled only through pulls on the goods it provisions across its borders. Either way the wire is one: a federation kept outside answers the boundary question as a **peer**, one pulled inside answers it as a scope; a centre goes in and a signed flow comes out, and where the blanket is drawn decides only whose fold the pull is the dual of (§If it solves globally, why federate?).

And **international solidarity has an operational form**: it is **acceptance-domain propagation across the border**. Credit earned under a plan is redeemable within that plan's acceptance domain (§Credits and acceptance domains); when a scope in one country and a scope in another **both accept** a plan, credit earned in the one becomes spendable in the other - and when the shared root endorses, the domain is the whole federation and the credit is **universally redeemable**, across the border as within it. Solidarity is then a **measurable reach**: the share of the planet's scopes inside one acceptance domain, the fraction of humanity whose labour earns credit good anywhere a fellow producer works. The same upward propagation of recognition that makes a credit federation-wide makes it **world-wide** - scope by scope, border by border, until the acceptance domain is the species.

[88] This is the international form of the move the document makes against **faux frais** and the metabolic rift: the distortions imperialism works through the world market - unequal exchange, value transfer, the development of underdevelopment - are properties of the commodity form, met not by a corrective accounting but by the form's supersession. What survives is uneven productive forces, a problem of **expanded reproduction** (Marx, *Capital* Vol. II) and worst-off-sensitive distribution, not of exchange. The membrane with the still-commodity world (§Credits and external liquidity) is the one place a world-market price - distortion and all - is paid, and there the commons absorbs it inward as SNLT of acquisition, the boundary sovereign.

And one thing the three partitions set free is the **nation itself**. Fused to territory and jurisdiction, a nation is a border to be drawn; separated from them, it is what it always was underneath - a **community of language, memory, and culture** - which the federation can carry as a **non-territorial scope**: an association one affiliates with from anywhere, reproducing its own schooling, archive, and tongue on the communal rail, its voice grounded like any scope's in the labour its members actually contribute to it. A dispersed people governs its own cultural reproduction without holding a hectare of ground, and one territory hosts many such communities without partition - the standing of the nation detached from the soil it was made to require.[89]

The border is the last duress-gap. Set the three specs beside it (§Dual power): the **legal** spec L is the border control the state imposes; the **internal** spec I is whatever genuine safety the federation reproduces of its own accord - a phytosanitary check it would run regardless, a skill-conformance the act needs anyway; the **desired** spec D is **free association and free flow**, the border the federation wants, which for the bulk of crossings is **no border at all**. The redundant core $L \cap D \cap I$ is the safety the federation already verifies and may demand the state's check wither into; the duress-gap $L \setminus D$ is the customs wall, the migration bar, the tariff that exists only because the law imposes it, advocated toward D. The deepest international reform is, exactly as for social property, a **transform-law**: the legal recognition of **plan-coordinated production across jurisdictions** - free movement of associated producers, mutual redeemability of credit, shared custody of transboundary agents - after which the border, like the ownerless mask, withers. The masks were the workaround while the law forced a name; the border-wrappers are the workaround while the law forces a line. The federation outgrows both the same way - by building, internally and in the open, the very coherence the border pretends to provide, until the political boundary is a vestigial shell around a metabolism that has already closed across it.[90]

[89] This is the kernel of the Austro-Marxist **national-cultural autonomy** of Bauer and Renner - nationality as a personal, non-territorial association governing its own cultural and educational life - which the framework realises directly: a non-territorial scope is exactly such an association, its membership and voice carried in the person-anchored credential substrate (Technical appendix) rather than in a place. The **territorial** definition of the nation - common territory, language, economic life at once - is the one fusion the architecture declines.

[90] The international duress-gap is the same reform diff of three spec-sets as §Dual power and §Social property, recognized, now per **binding border crossing**: L from the jurisdiction registry, I from the scope's own audit layer, D governance-declared as free association. Its $L \setminus D$ region - the customs wall, the migration bar - is the **transform-law** target whose D is recognised cross-jurisdiction social plan, won only by political power and solidarity, never by self-verification. See `module/playnet/planner/legal/reform.scm`.

12. Appendix: the fragility catalogue — assessed figures (June 2026)

The federation's possibility-tensions are not engineering constants. **Which** fragilities it tracks, where it places each tipping point, how steeply the curve rises — these are political-economic choices, custodied by a `tension-registry` and ratified at the UC level. But governance amends something **concrete**, so the registry is seeded with the real figures the Earth-system sciences report. This appendix records that seed **as assessed in June 2026** — dated deliberately, because these numbers move: the planetary boundaries now carry an annual Planetary Health Check, and the tipping-point thresholds are revised as the models sharpen. What follows is a starting estimate to be ratified and kept current, never a standing truth.

Two independently-developed frameworks seed it — and, the white paper's claim made good, both reduce to the **one** fragility profile $\varphi(s) = -w \log s$ at different settings of its dials (the range, the precaution placement α, the steepness w, the time envelope, the coupling graph). A planetary **boundary** is a precautionary setpoint on a control variable with a designed gradient of increasing risk; a climate **tipping point** is a physical bifurcation threshold carrying an uncertainty range. Same wall, different placement.

12.1. The nine planetary boundaries

The safe-operating-space framework of the Stockholm Resilience Centre and the Potsdam Institute. As of the 2023 assessment, **six of nine are transgressed**; the 2025 Planetary Health Check moves ocean acidification across the line, for seven. The boundary value is itself precautionary, so each enters as a frag with α spending the **designed** buffer (recoverable flows central, $\alpha \approx 0.5$), not stacked with a second margin.

process	control variable	boundary	current	status
climate change	atmospheric CO_2 (ppm)	350	417	transgressed
biosphere integrity	HANPP (% Holocene NPP)	10	30	transgressed
land-system change	forest cover (% potential)	75	60	transgressed
freshwater change	blue / green (% land area)	10.2 / 11.1	18.2 / 15.8	transgressed
biogeochemical — N	N fixation (Tg N/yr)	62	190	transgressed
biogeochemical — P	P to ocean (Tg P/yr, global)	11	22.6	transgressed
novel entities	untested synthetics released	0	*released*	transgressed
ocean acidification	aragonite saturation Ω_{arag}	2.75	2.80	at margin (2025: 2.86 / 2.84, transgressed)
atmospheric aerosols	interhemispheric AOD diff.	0.10	0.076	within (regional exceed.)
stratospheric ozone	O_3 column (DU)	276	284.6	within

Figure 33: **Nine planetary boundaries.** Source: Richardson, Steffen, Rockström et al., *Earth beyond six of nine planetary boundaries*, Science Advances 9 (2023), Table 1; ocean-acidification update from the PIK Planetary Health Check 2024/2025. Boundary values where retained from Steffen et al. 2015 / Rockström 2009. The federation draws **directly** on N and P (per-period rate boundaries); climate enters as a cascade (below); the remaining processes are witnessed, not yet drawn.

12.2. The sixteen climate tipping elements

The bifurcation thresholds of the Global Tipping Points programme. Because crossing is self-amplifying and often **irreversible**, each is placed at $\alpha = 0$ — the lower, most-cautious end of its range. The headline: at ~ 1.1°C of warming already reached, **five elements** (Greenland and West Antarctic ice, Labrador-Sea convection, low-latitude coral, abrupt permafrost thaw) lie within their threshold range; the rest become likely through the Paris range and beyond.

tipping element	min	central	max	character (°C)
Greenland Ice Sheet	0.8	1.5	3.0	irreversible/committed; 10 kyr
West Antarctic Ice Sheet	1.0	1.5	3.0	irreversible; 2 kyr
Labrador–Irminger / SPG conv.	1.1	1.8	3.8	abrupt (decadal), hysteresis
Low-latitude coral reefs	1.0	1.5	2.0	fast; abrupt mass mortality
Boreal permafrost (abrupt thaw)	1.0	1.5	2.3	abrupt local thaw feedback
Barents Sea ice (abrupt loss)	1.5	1.6	1.7	abrupt; narrowest range
Mountain glaciers	1.5	2.0	3.0	largely irreversible
Sahel / W-African monsoon	2.0	2.8	3.5	abrupt greening shift
East Antarctic subglacial basins	2.0	3.0	6.0	irreversible; slow
Amazon rainforest dieback	2.0	3.5	6.0	largely irreversible; century
Boreal forest (S. dieback)	1.4	4.0	5.0	gradual dieback
Boreal forest (N. expansion)	1.5	4.0	7.2	gradual expansion
Atlantic overturning (AMOC)	1.4	4.0	8.0	abrupt; widest range
Boreal permafrost (collapse)	3.0	4.0	6.0	carbon release 175–350 GtC
Arctic winter sea ice	4.5	6.3	8.7	abrupt; low hysteresis
East Antarctic Ice Sheet	5.0	7.5	10.0	irreversible; very slow

Figure 34: **Sixteen climate tipping elements**, threshold in °C of global warming above pre-industrial. Source: Armstrong McKay, Staal, Abrams, Winkelmann, Sakschewski, Loriani, Fetzer, Cornell, Rockström & Lenton, *Exceeding 1.5 °C global warming could trigger multiple climate tipping points*, Science 377 (2022), Table 1; carried into the Global Tipping Points Report 2023. Red marks the five already within range at ∼ 1.1°C. "central" is the authors' best estimate, the min–max their assessed uncertainty, not a confidence interval.

12.3. How they enter the plan

Climate is encoded as a cascade, not a flat cap: the federation draws one flux — co_2 (its fair **share** of the global remaining budget) — and the **same** draw stresses two driven states through the coupling graph. $co_2 \rightarrow$ warming (°C, through the transient climate response, 0.45°C per 1000 $GtCO_2$) carries the tipping cluster; $co_2 \rightarrow$ ocean acidification (Ω_{arag}) carries the aragonite boundary. The aragonite wall is the **tighter** of the two — it binds at ∼ 185 $GtCO_2$ where the warming wall would allow ∼ 444 — so the same emission meets two planetary limits at once, the nearer one first. A reassuring cross-check falls out: the transient-response conversion places the 1.5°C cluster at ∼ 444 $GtCO_2$, squarely inside the IPCC AR6 remaining budget of 400–500 — the two frameworks agree on the number.

The keeping of the figures honours the **non-control** caveat: the global commons are not the federation's to set, so a "budget" here is its fair share and beyond it the federation bears witness rather than pretending to a lever it does not hold. The whole catalogue is **draft** — every cell above is amendable data in the `tension-registry`, sourced and dated so a later assessment can supersede this one without touching a line of solver code.[91]

[91] Encoded in `module/playnet/planner/knowledge/catalogues/eco.scm`; custodied by `tension-registry.scm`; the two-frameworks-one-frag reduction is argued in `docs/planning/solve/possibility-tensions-unified.md`. Figures transcribed verbatim from the cited primary sources, June 2026.

13. Glossary

Agent

A person, organisation, or ecological entity with economic agency (VF). Agents create, transform, and exchange value, and make agreements with each other. The federation, its scopes, and its citizens are all agents.

Ecological agent

An ocean, a forest, a watershed, a soil microbiome - a natural system recognised as an agent (VF) that produces through its own dynamics. Every scope that draws from natural systems holds custody and carries ecological specifications (regenerative envelopes, absorptive capacities) in its plan. The VF contained-in relationship models nested ecosystem structure. Stewardship is administration applied to natural systems: the same verification mechanism checks the draw against the specification. Outputs are observed abundance recorded as economic events.

Regenerative envelope

The rate at which a natural system replenishes what is drawn from it. Enters the plan as a **possibility-tension** (a hard barrier): the draw's **pull** rises continuously and diverges as it nears this rate, so the envelope is never crossed. The complementary measure is **absorptive capacity** - the rate at which a natural system can process what is released into it.

Economic resource

An observable instance of a resource specification (VF). Resources are incremented by *produce* events and decremented by *consume* events; a count that finds the ledger off corrects it by a *raise* or a *lower* of the difference, as the ledger stood at the instant of the count. A resource can be inventoried (tracked by quantity) or non-inventoried. Economic resources are the leaves of the treemap - the actual stock of things the federation reproduces.

Resource specification

The lowest-level type of an economic resource (VF). A resource has exactly one specification, which defines what kind of thing it is. Leaf cells in the treemap group economic resources by their specification; the spec is the schema, the resources are the substance.

Resource classification

Taxonomic categories used to filter, match, or group economic resources (VF). They nest arbitrarily (food → grain → bread). Shown as header bands in the scope-level treemap.

Resource quantity q_r

The quantity of resource r reproduced this period, in natural units (loaves, litres, kg, procedures). Times direct labour $\mathrm{SNLT}_{\mathrm{direct}(r)}$ it gives the cell's area; times embodied $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$ it gives the unit's labour-cost for credit.

Reproduction labour R_r

The **direct** living labour spent this period to (re)produce resource r at its own stage, $R_r = q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}$ - the cell's area on the surface (the figures and captions call this **planned labour** - the same quantity, named from the plan's side; summed across all cells it is total labour L, each hour counted once where it was performed). The embodied cost $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$ is the separate measure (credit, sizing). Decomposes by intent into the three deductions: **maintenance** (replace what is used up, holding the target steady - simple reproduction), **expansion** (raising the target and capacity - expanded reproduction), and **insurance** (the reserve fund against accidents and calamities - sized by the **computation of probabilities**, **not** equity, and so a hard possibility-tension - unlike the soft, wanted buffer of the same target). A gap opens only because

the level fell (maintenance) or the target rose (expansion); summed across the surface, the three are readable as funds.

Desired quantity q_r^* and target The desired quantity is the **soft** level the plan drives to: enough to meet ongoing drawdown, plus a **smoothing buffer** against its variability. The buffer is **part of desire** - we want the level - so it is soft, traded in harmony (filled from slack, the first to yield under contention); its rate b_r is sized in *Desire on the turn*. Separate from it, the **insurance reserve** $q_r^{\text{res}} = \iota_r q_r^*$ - the third deduction, sized by the **computation of calamity probabilities**, "not calculable by equity" - is held as a **hard** possibility-tension that out-pulls desire yet yields to true calamity.[92] The **target** on the cell's tab is their sum, $q_r^* + q_r^{\text{res}}$ - the full standing level on hand; its distance from the current on-hand quantity summons the period's reproduction labour. The credit pool draws on q_r^* alone, the target net of the held-back reserve. Services carry no target: they hold nothing on hand.

Stock / service A **stock** persists on hand (tools, trained people, durable and buffered goods), carries a target, is drawn down by use and replenished by reproduction, and its level bounds the labour that draws through it (its capacity role). A **service** persists nothing - it is reproduction labour consumed in the moment it is produced (a consultation, a lesson, a verification), set directly by demand and bounded by the stocks it draws on. The two are ends of one holding-time spectrum.

Skill set S_p The set of skills citizen p can perform, drawn from the federation's skill catalogue $\mathcal{K}$. In VF, each skill is identified by a resource specification with the *work* action.

Offered availability A_p A citizen's declared time windows, intensities, and spatial catchments - when, where, and how much they can work. The federation's total pool is $A = \bigsqcup_p A_p$.

Scope A branch of productive activity (food, manufacturing, education…) with its own governance. Each scope s carries a total $L_s = \sum_p L_{p,s}$. Scopes form a tree; the Universal Commune is the root.

Universal Commune (UC) The root of the scope tree, formed at planning-time. Its children are all federation roots. The UC holds the whole pool of offered availability and coordinates federation-level functions: plan endorsement, acceptance-domain propagation, and independent verification.

Acceptance domain $\text{accepted}(P)$ The set of scopes that have accepted plan P. Credits earned under P are redeemable within this domain. Acceptance propagates monotonically upward;

[92] Realized as a floor WITH A MARGIN in the good's reproduction band - `#:insurance` on `plan-from-records`, or a `<fragility>` whose side is `'floor` (`floor-on-good`, `engine/fragility.scm`) - per located good a profile (lo, hi, α, w) poured into the planner's `held` term (`*band*`, `build-located` "REPRODUCTION BAND"): divergent at lo, so the calamity stock is never breached, satiating at the margin $\{\text{quantile}\}_\alpha[lo, hi]$, so nothing is over-made for it. It is applied per scenario in the robust assembly (`engine/robust.scm`) - full margin in the nominal year, margin eroded and lo intact in the drought - and it **yields to true calamity**: by default the reserve is not asked of the futures at probability zero, the precautionary tail beyond the record, so the plan banks what feeds the famine rather than a reserve to outlast it (`reserve-yields-to`; on the Doon record strictness would bank a quarter-season more, unspent). In one world there is no tail to name - a shock arriving now is a program with no answer - and `reserve-dilemma` prices the two ways out as `plan-dilemma` does a floor against a wall: hold the reserve and go short, or eat the reserve and be fed, the one scarcity read from either side (`tests/planner/test-reserve-dilemma.scm`). `#:reserve` is the soft buffer beside it (a low-weight desire tranche, shed first). `tests/planner/test-insurance-reserve.scm` runs the three shapes - bar floor, soft buffer, `held` - at one number: the buffer starves under scarcity, the floor sits at bare need with no margin, `held` holds the margin and erodes to lo; and across two futures the buffer plants all the high-yield crop and starves in the drought where `held` plants the mix that keeps lo in every year.

when the UC endorses, the domain is the whole federation and the credit is universally redeemable.

Action — A verb on a flow that defines what happens to a resource (VF). Core actions: *produce* (create a resource, incrementing its stock), *consume* (use it up - destroyed, decrementing its stock: flour into bread), *use* (employ it without destroying it - a tool persists but wears, and the wear is its depreciation), *work* (apply labour), *transfer* (move custody or claim - a claimable good leaving the pool to a citizen), *cite* (reference without drawing down). The *consume*/*use* distinction is what separates circulating means (used up each cycle) from fixed means (used across many cycles, depreciating).

Process — An activity that transforms inputs into outputs (VF). Outputs may become inputs to other processes, forming chains and cycles.

Recipe — A reusable template for producing or maintaining a resource (VF). Contains the input–process–output graph: bills of material, routings, and labour requirements. A resource may carry both a production recipe and a maintenance recipe; each generates its own work-intents. The bread DAG in Figure 2 is a recipe.

Value rollup — Recursive summation across the process tree of a recipe (VF); a recipe with several co-products splits its one cost among them by relative standalone $\mathrm{SNLT}_{\mathrm{crystallized}}$, the shares summing to the whole. Used to compute the embodied cost $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$: the direct labour $\mathrm{SNLT}_{\mathrm{direct}(r)}$ at this stage, plus the $\mathrm{SNLT}_{\mathrm{crystallized}}$ of every input **consumed** (each times the quantity used), plus the depreciation of every means of production **used** (its wear δ per use times **that means'** $\mathrm{SNLT}_{\mathrm{crystallized}}$). Depreciation is the sole channel by which a means enters the product: **repair** is direct labour ON the means - it raises the means' own $\mathrm{SNLT}_{\mathrm{crystallized}}$ and reaches goods through depreciation, never as a separate term in their cost. A **contained** resource adds its own $\mathrm{SNLT}_{\mathrm{crystallized}}$ separately - the containment cascade events each draw-down, so a loaded container's value is its own assembly plus its contents, and the two never double-count. Note the rollup yields $\mathrm{SNLT}_{\mathrm{crystallized}}$ (the embodied measure), not the surface area, which uses $\mathrm{SNLT}_{\mathrm{direct}(r)}$ alone.

Socially necessary crystallized labour time $\mathrm{SNLT}_{\mathrm{crystallized}(r)}$ — The **total** labour congealed in one unit of resource r: the direct labour $\mathrm{SNLT}_{\mathrm{direct}(r)}$ plus, by value rollup, the crystallized labour of inputs consumed and the depreciation of means used. The crystals of social substance congealed in the product - its total value. Socially necessary, exactly as the direct measure is - the two differ as congealed labour to living, not as necessary to incidental. It sizes work-intents and settles the credit drawdown ($K = \sum_r q_r^* \cdot \mathrm{SNLT}_{\mathrm{crystallized}(r)}$) - transparent bookkeeping for distribution, never an exchange-value. It is **not** the surface area: cell areas use the direct measure $\mathrm{SNLT}_{\mathrm{direct}(r)}$, so they sum to total labour without double-counting. A resource may carry distinct values for maintenance, expansion, and insurance.

Socially necessary direct labour time $\mathrm{SNLT}_{\mathrm{direct}(r)}$ — The **living** labour socially necessary at resource r's **own** stage of production - the baking in a loaf, the milling in flour - per unit, excluding the crystallized labour of its inputs (counted in their cells). The surface's per-unit measure: $\mathrm{area}(r) = q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}$, and $L = \sum_r q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}$. The labour newly added, against the total value of $\mathrm{SNLT}_{\mathrm{crystallized}}$.

Relocation (transport & storage) One recipe shape, not two mechanisms: *pickup* a good, *dropoff* it, *use* a container, *work* a driver. Tiled over the spacetime lattice's arcs - a **spatial** arc (place⟶place, scaled by the measured arc: trip time for *work* and *use*, road distance for fuel and loss) is transport, a **temporal** arc (period⟶period, scaled by one period) is storage. Which arc is **derived from the container**, never marked on the recipe. Loss in transit or store (spoilage, breakage) is a *consume* on the recipe of the good's own lost fraction - the same shape as a tool's wear, sized by the manner, not stamped on the good. Not *faux frais*: a real social cost of a distributed society.

Measured arc (arc-time table) The spatial arc's real cost, keyed by (from, to, **manner**, period-class): the federation's self-hosted router prices every pair of located places per manner of moving (its **prior** - trip time and road distance off the commons' map and timetables), and every performed journey, filed under the mode it was made by (that mode recipe's costing is its manner), refines it (recency-weighted **measurement**, which speaks ahead of the prior). Rush-hour and weekend arcs keep separate entries. Absent both, plain geometry carries - the program is unchanged, only the numbers truer. The router knows the territory; the table is what the federation has learned of it.

Container A **used** resource that carries contents - a vehicle through space, a warehouse through time, a dwelling that shelters across both. Its **classification** (resource-ClassifiedAs) decides which: keep-in-place containers relocate their contents through time, mobile ones through space. "Stationary" is not an absolute - moving the warehouse itself is another recipe, in which the warehouse is the cargo. What it takes to move a thing lives in its classification, not in a flag.

Rider (mover) A person modelled as a good that travels the lattice: their located stock is a **presence**, and where it rests they **supply labour** (the presence gates the hours). They ride the same recipe as cargo, but can also **propel themselves** (walking = a mode whose only work is the rider's own). A travel **mode** is a recipe in the scope's store whose *work* flow resolves from its own *pickup*, instantiated per rider; modes (walk, cycle, ride) are gated by **skills**; one who cannot walk is carried - by the same logic a crate, lacking "walk", takes a vehicle. Planned-work transport is the rider's real social labour; a hailed ride is instead a means of consumption (a *deliver-service* good drawn in free time against credits). Every scope seeds the baseline modes (walk, bicycle, car); a journey is filed under the scope's mode for the way it was made, and the rider's own hours on a planned ride are a *work* commitment. Cargo is a mover without the labour.

Intent A potential future event not yet agreed to (VF). Work-intents demand a skill k at time t in catchment c. Commitments satisfy intents once agents agree.

Work-intent An intent with action *work*: a demand for skill k at time t in catchment c. Matched when $k \in S_p$ and the citizen's offered availability intersects.

Commitment A potential future event that agents have agreed to pursue (VF). The plan's scheduled work. Economic events fulfil commitments.

Accept (decided column) A person's acceptance of a work or material offer, entered into the solve as a **decided column** - a fact identical in every scenario, substituted out of the program through the discrete search's own door, refused if it starves a floor in any future. The **hold** is the rung before it: an open offer riding every re-solve as a wall until its window's forcing point. The horizon gate derives how far ahead an

accept may reach (the booking-augmented critical path) and how soon it must resolve (§The accept: a person decides a column).

Recipe character — A recipe's measured conduct under stress: per flow of its own - its output, each input, the skill it works - the **curve** of **capture** (what a run made, of what its weather allowed: the day's sky on its output and the scarcest input that reached it) against the stress delivered there, read off witnessed runs and never stored. Stress is what a list of shocks does to the recipe's own flows, read alike for the sky a run lived under and the future a solve plans against, so a record meets every future that touches the recipe. Every future of a solve reads each curve at the stress it delivers to that flow, and the most adverse across them, so the plan leans on the recipe in proportion to what it delivers there; a flow the weather has yet to press reads the most adverse response the recipe has shown at that stress. The words - fragile (harm that accelerates), antifragile (a gain that accelerates), robust, in proportion, or **awaiting observation** under three stress levels - describe the curve's bend and steer nothing (§Character is a measurement).

Reach and margin — A recipe's **reach** is the heaviest stress its record has a run under, flow by flow: what it is proven to. Its **coverage** of a future is that reach over the stress the future delivers, at most one. In the future a floor is defended in without a forecast - the tail, at probability zero - a recipe yields by its coverage against the best-covered recipe for the same output, so needs rest on what has been proven and a trial takes surplus alone. A construction with no number in it. A recipe is **experimental** while every run was calm, **observed** after; a trial begins by a person's offer to work it, held as a floor on its column and refused where it would take a reproduction floor's means (§Character is a measurement).

Plan — A schedule of related processes with defined deliverables (VF). Generated from recipes by scaling to reproduction targets - the levels the federation aims to hold on hand - then adjusted for what is currently on hand. Reproduction labour decomposes into maintenance, expansion, and insurance. The plan is the live configuration of stock targets, the matching queue, and the commitment ledger. Figures 1–2 are plans rendered spatially.

Economic event — An observed past occurrence: a resource was produced, consumed, used, or transferred (VF). Events are immutable. For *work* events, matched hours enter L_p automatically when worked; output quantities enter the planning layer for next-cycle adjustment.

Matching — The assignment of citizens to work-intents in accepted plans. Two layers: **federation matching** (plans to labour - voice-weighted, optimises harmony subject to labour availability) and **egalitarian matching** (producers to slots - uniform preference weights). The matching is the sole accounting primitive: $L_{p,s}$ derives from it directly, and voice and credit follow.

Egalitarian matching principle — Within each accepted plan, producers are assigned to work slots from the subset of citizens who hold the required skill, with uniform preference weights $w_p^{\text{pref}} = 1$ - equal standing among all who can do the work; a citizen's own preference over their slots is equal and bounded, and moves nobody's total load. **How** the work is done is shaped by contribution; **what** the community needs is shaped by all three voice components; **who makes which part** is determined by citizenship.

Validated labour L_p

Matched hours worked. When a citizen is matched to a work-intent in an accepted plan and works the slot, those hours enter L_p directly. Revocable through ordinary scope self-governance for visible non-contribution. Federation aggregate: $L = \sum_p L_p$.

Plan progress

The ratio of **realized labour** to **planned labour** for a cell or scope: how much of the cell's matched hours have been worked. The dark fill from the bottom of each treemap cell is realized labour; the lighter zone above is unrealized labour. Equivalent to: how far through the planning horizon the scope has progressed.

Planned labour

The direct living labour budgeted for a cell on the surface, computed as $q_r \cdot \mathrm{SNLT}_{\mathrm{direct}(r)}$ - the cell's area on the treemap. Synonym for the cell's reproduction labour, named from the plan's side.

Realized labour

The portion of planned labour that has been worked - matched hours already converted into validated labour. The dark fill rising from the bottom of each treemap cell.

Unrealized labour

The portion of planned labour not yet worked - matched hours still in the future. The lighter zone above the realized fill in each treemap cell.

Your labour

The individual citizen's contribution within a cell - their own realized labour as a fraction of the cell's planned labour. Tracked per citizen and surfaced on the live individual-claimable cells; the printed surface (Figure 2) renders only the aggregate intent decomposition.

Producer voice $v^{\mathrm{prod}}_{p,s}$

Weight proportional to validated hours worked on the part an act concerns: $v^{\mathrm{prod}}_{p,s} = L_{p,s}/L_s$, the scope s the standing part and every finer part read by the same rule. Shapes **how** the work is done - stances on parts, recipe selection, plan verification, scope self-governance.

Part

A conjunction of nodes from the shared classifications - scope, process or recipe, means at the grain of its specification, place, window, skill, folksonomy tag - at most one node per kind, all holding at once. A part may narrow and may never join. It compiles at each assembly to the columns it covers, and it is what a stance covers, what a price is read over, and what hours are folded by.

Stance and hand

A stance is a number in $[-1, +1]$ on a part - **less of this, or more, for anyone** - weighed by the speaker's producer voice on that part. A person's hand is held hour by hour: over the parts any one of their validated hours falls in, portions sum to at most one, so each hour speaks at most once and rotation costs no voice.

Voice field τ

The bounded re-weighting a plan feels from the standing stances: $w'_j = w_j\big(1 - \tau s_j\big)$ with s_j the column's push read against its good and clamped to $[-1, 1]$, τ a governed bound below one (default zero). A push shared by every way of a good cancels; felt hours equal clock hours at the worked mix; walls stand untouched.

Displacement of voice

What the standing plan moves against the same program at zero field, at one settled floor; a part's marginal displacement $-\tau \sum_{j \in \text{ part}} \ell_j x_j$, additive across parts. Shown to those holding the stance and to everyone the cost falls on.

Contrary observation

A signed constative act by a worker of a part: a mapped figure - a yield, an input rate, a step's hours - against a reading their own work event witnessed. It

moves the figure in the cautious direction alone, over every scenario, one live per observer, part and action; the nominal figure stands until the map is corrected.

Consumer voice v_p^{cons}

The citizen's weight for consumption decisions and their share of the individually-claimable pool. A blend of three components: contribution ($\propto L_p/L$), solidarity ($\propto 1 - a_p$), and citizenship ($= f/N$). Weights sum to 1. Claim capacity is $W \cdot v_p^{\text{cons}}$; drawable credit is that less the drawdown. Accrues through participation; converges to $1/N$ as $f \to 1$.

Means of production

Tools, infrastructure, materials, and facilities that remain social property - a stock held on hand whose level bounds the labour every recipe draws through it. Reproduced by maintenance, expansion, and insurance.

Skills

The federation's **capacity stock of skilled population**, indexed by skill $k \in \mathcal{K}$ - the (re)production of labour-power, rendered as the catalogue of skills sustained at desired levels. The natural unit is **number of people trained in k**. Each cell's area is $q_k \cdot \text{SNLT}_{\text{direct}(k)}$ (the direct labour of training and sustaining), while the embodied cost $\text{SNLT}_{\text{crystallized}(k)}$ - rolling in materials and facilities - serves sizing and credit. Reproduced by **maintenance** (recertification and refresher training, plus forming new people to replace those who leave - holding the count), **expansion** (forming people to grow the count beyond its current level), and **insurance** (the redundancy margin held on hand against loss, sized by computation of probabilities). The trained person is the stock, and its level bounds the labour of every recipe requiring k; their exercise of the skill appears as work flows in recipes throughout the surface.

Means of consumption

The share of the social product destined for personal or communal use - both stocks (durable and buffered goods) and services (consumed as produced). Distribution runs along a separate axis from what the product is: two rails, each carrying goods and services alike.

Individual claimable

Goods or services drawn against personal credit (a loaf, your own consultation). Distributed via the credit identity, through a **claim** recipe; a good is on this rail because such a recipe consumes it for someone.

Claim

A recipe, one per citizen and good: `consume` the scope's good and the citizen's credit at the good's $\text{SNLT}_{\text{crystallized}}$, `produce` the citizen's own (`bread~ada`) - custody moving because the flow says so. A planned claim is a transfer commitment; the citizen's confirmation records the transfer event against it, settling the drawdown. What a process `uses` - a means - is never claimed.

Communal

Goods or services allocated by social decision and consumed in common (public shelter, a vaccination campaign, schooling) - distributed through collective governance.

Administration

The labour of answering: does the intent match the specification? Two levels: **scope-level self-governance** (plan formation, coordination, standard compliance, handling non-contribution - budgeted within each scope's plan, performed by the scope's own producers) and **UC-level verification** (independent review of whether ledger entries correspond to reality - matched from the federation pool by EMP, independent of the scope being verified). Administration is itself productive activity with its own recipes; the concrete needs of production determine its shape and timing. Minimised by design.

Social desire w_d^{desire}

The federation-level demand signal: individual priority weights weighted by consumer voice and aggregated. $w_d^{\text{desire}} = \sum_p v_p \cdot \hat{x}_p(d)$. The planner generates a concrete plan to satisfy it; crystallized SNLT is derivable from that plan.

Priority weights $\hat{x}_p(d)$

Citizen p's relative priority on demand-dimension d, computed as the product of weights along the resource classification path: $\hat{x}_p(d) = \prod_{i \in \text{ path}(d)} w_{p,i}$. Unit-sum at each branching point. On the individual-claimable rail, these shape the citizen's order book; on the communal rail, they carry the citizen's voice into collective allocation.

Harmony

The concord the plan maximises: the well-tempered whole that relaxing tension achieves, measured as tension resolved, $\mathcal{H} = -\sum_d D_d - \sum_a P_a$. Tension well-resolved - metabolised, relaxed into harmony moment to moment. Owed to Cockshott and Cottrell, *Towards a New Socialism* (1993).

Tension

A continuous, labour-time measure of a configuration's distance from where the federation wants to be; the plan **relaxes** it, and **harmony** is the concord relaxation achieves. Two species: **desire-tension** (soft) and **possibility-tension** (hard).

Relaxation

The resolving of tension - the metabolism's continuous work. A tension relaxes as a desire is met or a draw eases off a limit; the plan never removes tension, only relaxes it toward harmony, beat by beat.

Desire-tension D_d

The soft species, in its shared-dimension member (a citizen's own want enters through the **cap** member beside it): an unmet final want, $D_d = w_d^{\text{desire}} \cdot g_d^2$ (squared gap, weighted by social desire). **Bounded**, so desires trade against one another. Pulls allocation toward producing more. The realm of freedom.

Cap (desire member)

The desire term a citizen's **own** stated want enters through: value $w/(1 + s/c)$ - bounded, and reading nothing the citizen reports - with the asked quantity standing beside it as a wall, $s \leq q$, and c the spec's reference scale (a catalogue fact). At the optimum the want is served to the lesser of its ask and its **water level**, the level the citizen's weight commands against the good's scarcity. Any rewrite of one's own asks is weakly dominated, machine-checked (*Honest by shape*); shared dimensions carry the fractional-gap quadratic instead.

Honesty radius

Per solve, off the certificate's strong-convexity modulus σ: the exact bound $2L_\Phi W/\sigma$ on how far a citizen's own weight can move the plan, scaled by their voice - L_Φ the term map's own Lipschitz constant, $1/c$ at the cap shape's catalogue scale c (one at the unstated default). Shown beside the declaration-time price, alongside the same σ's reading of how far the solved-to-tolerance plan already sits from the exact one.

Possibility-tension P_a

The hard species: the strain of pressing a limit of the possible (labour endurance L/A, capacity, skill redundancy, reserves, ecological envelopes, care). A barrier that **diverges** at the limit - that divergence is its **fragility** - so near the limit it out-pulls every desire and is never crossed. Pulls allocation away from the edge. The realm of necessity.

Pull

How hard a tension pulls toward relaxation - the marginal harmony that relaxing one more unit of it would yield, the local slope of its curve (formally, the dual variable; we drop "shadow price" since there is no price and nothing is imputed).

	The plan is the configuration where every tension pulls equally hard. Realised metabolically as the **loudness** of a gap propagating upstream.
Gap g_d	Fractional shortfall on demand-dimension d: 0 = fully met, 1 = wholly unmet. Enters the harmony score quadratically.
Satisfaction level σ_p	Priority-weighted need-fulfilment for citizen p, in $[0, 1]$.
Social desire satisfaction Φ	Harmonic mean of individual satisfaction levels: $\Phi = N / \sum_p (1/\sigma_p)$. Structurally sensitive to whoever is worst off.
Individually-claimable pool $\mathcal{R}$	The resources set aside for personal consumption - the goods with a claim edge, the set the plan publishes from its claim recipes. Labour-content draws on the quantity standing above each good's social portion s_r - what the scope's own processes consume this horizon plus the held-back insurance reserve: $K = \sum_{r \in \mathcal{R}} \max(0, q_r - s_r) \cdot \mathrm{SNLT}_{\mathrm{crystallized}(r)}$. Releasing a reserve lowers s_r and grows K.
Capacity to labour a_p	A citizen's capacity, in $[0, 1]$, attested by fellow workers through capacity-recognition (a production-governance act, excluding self-attestation, weighted by labour-voice). $(1 - a_p)$ measures reduced capacity.
Drawdown	The running accounting of how much of the citizen's claim capacity $W \cdot v_p^{\mathrm{cons}}$ has been claimed from the individually-claimable pool - read off the plan per claim (`credit-drawn`), settled as each claim is confirmed. Consumer voice itself is unchanged by drawdown.
Entitlement pool W	The labour-hours the citizens' shares are shares of: the pool as it stands, K, together with everything already drawn from it, $W = K + \sum_p d_p$. A claim moves goods from the pool to the claimant and leaves W where it was, so drawable credit $W \cdot v_p^{\mathrm{cons}} - d_p$ sums to exactly K: the credit left is the goods left.
Desired inventory	The citizen's live order book - priority weights projected onto the individual-claimable pool. Fill shows what has been delivered.
Stock	Produced goods not yet claimed or distributed. The dark-green remaining portion in individual-claimable cells.
Social drawdown	What harmony has distributed to other citizens from the available stock.
Productive forces	The combination of means of production, labour-power, skills, scientific knowledge, and social organisation that determines how much the federation can produce with a given amount of labour. As productive forces grow, the same output requires less labour-time, and the social free time f rises.
Real wealth	Disposable time outside that needed in direct production, for every individual and the whole society. As productive forces grow, the creation of real wealth depends less on labour-time and more on the general state of science and its application to production. Its foundation is the development of the social individual, which both produces and is produced by free time. The social free time f measures this directly.
Social free time f	Fraction of total social time the solved plan leaves unplanned: $f = 1 - \Lambda/(NT)$, with Λ the hours the plan places (making, moving, training), published after

each solve. Validated hours set shares and never f. As productive forces grow, f rises toward 1.

Social play time $\mathcal{P}$ — The joint measure of free time and equal social desire satisfaction: $\mathcal{P} = \min(f, \Phi) \in [0, 1]$. $\mathcal{P} \to 1$ only when both $f \to 1$ (most of social time is free) and $\Phi \to 1$ (every citizen's needs are fully and equally satisfied). The limit toward which the federation moves.

Metabolic coherence $M(s)$ — Ratio of a scope's internal flows to total flows: $M(s) = F_{\text{internal}}/F_{\text{total}}$. Two coupled axes: **labour metabolism** (denominated in crystallized SNLT) and **material metabolism** (substance-denominated). Neither subsumes the other.

14. References

> *"Let us now picture to ourselves, by way of change, a community of free individuals, carrying on their work with the means of production in common, in which the labour power of all the different individuals is consciously applied as the combined labour power of the community.*
>
> *The products of labor are, in general, utilities - objects of use. In a community of associated producers, labor manifests itself as simple, useful labor -'simple' in the sense that its ultimate end is the production of use-values, not exchange-values. This labor is regulated by the social aim it serves. The socially necessary labor-time for the various products is determined by the technical difficulties of production and by the community's collective skill in overcoming them. Finally, there is a definite relation between the community's different branches of activity - hunting, fishing, tool-making - and the total labor-time allotted to each. The community keeps accounts of its inventory of use-values, the various operations of its labor, and the time it takes to produce them.*
>
> *The total product of our community is a social product.*
>
> - *One portion serves as fresh means of production and remains social.*
> - *Another portion is consumed by the members as means of subsistence.*
>
> *A distribution of this portion (the means of subsistence) amongst them (the community) is consequently necessary.*
>
> *The mode of this distribution will vary with the productive organisation of the community, and the degree of historical development attained by the producers.*
>
> *We will assume, but merely for the sake of a parallel with the production of commodities, that the share of each individual producer in the means of subsistence is determined by his labour time.*
>
> *Labour time would, in that case, play a double part:*
>
> - *Its apportionment in accordance with a definite social plan maintains the proper proportion between the different kinds of work to be done and the various wants of the community.*
> - *On the other hand, it also serves as a measure of the portion of the common labour borne by each individual, and of his share in the part of the total product destined for individual consumption.*
>
> *The social relations of the individual producers, with regard both to their labour and to its products, are in this case perfectly simple and intelligible, and that with regard not only to production but also to distribution."*
>
> — *Capital* Vol. I, Ch. 1 §4 (paraphrased)

1. Paul Cockshott and Allin Cottrell, *Towards a New Socialism* (1993). The harmony objective function, the computational reply to Hayek's knowledge problem, and labour-time accounting as a basis for socialist planning.
2. Tjalling Koopmans (ed.), *Activity Analysis of Production and Allocation* (1951). The technology matrix, activities and their levels, and the efficiency-price duality.
3. Leonid Kantorovich, *The Best Use of Economic Resources* (written 1942; published 1959). The plan as a linear program, its duals as objectively-determined valuations. Appendix I carries this design's constraint side nearly symbol for symbol: time-indexed goods, methods as columns, intensity vectors, zero-profit valuations on methods in use, transport potentials and line rents, and the rate of return on investment as a ratio of period multipliers. The 1975 Nobel, shared with Koopmans; the Newton-Kantorovich theorem - the convergence theory of the second-order method that now solves his problem - is also his.
4. János Kornai and Tamás Lipták, "Two-Level Planning", *Econometrica* (1965). Centre and sectors iterating through the duals - the ancestor of the scope decomposition.
5. Oskar Lange, "The Computer and the Market" (written 1965; published posthumously 1967, in Feinstein, ed., *Socialism, Capitalism and Economic Growth: Essays Presented to Maurice Dobb*). Market socialism's author concluding the simultaneous equations belong on a computer.

6. Benjamin Peters, *How Not to Network a Nation* (2016). The history of Glushkov's OGAS, the Soviet real-time economic network.
7. Eden Medina, *Cybernetic Revolutionaries* (2011). The history of Beer's Cybersyn, Chile's real-time economic feedback layer.
8. Francis Spufford, *Red Plenty* (2010). The literary account of the optimal-planning moment.
9. ValueFlows, *A vocabulary for the distributed economic network* (ongoing). valueflo.ws. The REA (Resource, Event, Agent) ontology extended for distributed economic networks: resource specifications, classifications, recipes, processes, plans, intents, commitments, economic events, actions, and the value rollup.
10. Carol Ptak and Chad Smith, *Demand Driven Material Requirements Planning (DDMRP)* (2011). Strategically-sized buffers at decoupling points, demand-driven replenishment, and buffer sizing by lead-time and demand variability.

15. About Playnet

Playnet is developed by a community of contributors through **participatory play labs** - collaborative sessions where the specification, the software, and the social plan are shaped together in practice. The document you are reading emerged from these sessions: each concept tested in conversation, each equation derived from the concrete needs of coordination, each design choice made by the people who intend to use it.

The project is an implementation: working software that any branch of productive activity can begin using today. Playnet is built on Spritely Goblins (distributed object-capability actors), GNU Guile Scheme, and the ValueFlows vocabulary. It runs on commodity hardware, communicates over mesh and overlay networks, and is free software.

The work proceeds from a simple conviction: the commons already contains everything it needs. The productive forces exist. The science of material metabolism is known. The means of computation are in every pocket. What remains is to build the social plan and invite people to use it. The private plans that currently organise production need only be outgrown.

playnet.earth